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Precise determination of gravity in geodetic networks in Slovakia – CBA2G approach

Pavol ZAHOREC[1,*], Juraj PAPČO[2], Miroslava JANČOVIČOVÁ[3], Ivan MARUŠIAK[4], Roman PAŠTEKA[5]

[1] Earth Science Institute, Slovak Academy of Sciences, Ďumbierska 1, 974 01 Banská Bystrica, Slovak Republic

[2] Department of Theoretical Geodesy and Geoinformatics, Faculty of Civil Engineering, Slovak University of Technology, Radlinského 11, 810 05 Bratislava, Slovak Republic

[3] Geodetic and Cartographic Institute, Bratislava, Chlumeckého 4, 827 45 Bratislava, Slovak Republic

[4] G-trend Ltd., Budatínska 16A, 851 06 Bratislava, Slovak Republic

[5] Department of Engineering Geology, Hydrogelogy and Applied Geophysics, Faculty of Natural Sciences, Comenius University, Mlynská dolina, Ilkovičova 6, 842 15 Bratislava, Slovak Republic

Abstract: We present software for calculating gravity acceleration at points of levelling networks in Slovakia. The chosen approach uses an existing detailed gravimetric database and a back-calculation (reconstruction) of the gravity acceleration from the Bouguer anomaly map. We analyse the accuracy of this approach on a sample of independent geodetic points as well as in-situ control measurements.

Key words: gravity acceleration, complete Bouguer anomaly, levelling networks, ellipsoidal heights

1. Introduction

The modern realization of the height system in Slovakia is based on the calculation of geopotential numbers. However, the problem is the lack of measured gravity at levelling points, less than 10 percent of points within Slovakia, according to *Majkráková et al. (2016)*. These authors compared different methods of calculating gravity at levelling points where measured data are missing. The most suitable method was the back-calculation me-

*corresponding author: e-mail: zahorec@savbb.sk

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thod from the complete Bouguer anomaly (CBA) map, i.e. the so-called CBA2G (Complete Bouguer Anomaly to Gravity) approach, where the exact position and height of the levelling points also play an important role. The first version of the software solution for this approach was created in 2015 (*Marušiak et al., 2015*) and reflected the current state of gravimetric database in Slovakia. However, over the last 10 years, the situation has changed in several ways. Currently, we have a larger amount of gravimetric data available, and a new generation digital elevation model (DEM) based on airborne laser scanning (LiDAR) is also available in Slovakia. This allows us to recalculate the gravimetric database and construct a newer, more accurate version of the CBA map. In connection with this, there was also a need to update the input data for the CBA2G approach, i.e. the CBA map and individual DEMs. In addition, a new implementation of the EVRS height system (EVRF2019) is currently valid in Slovakia, as well as the new quasigeoid DMQSK2024-E (*Jančovičová et al., 2025*), which is another reason for updating the CBA2G approach.

2. Gravimetric database and CBA map of Slovakia

The first version of the CBA2G software solution was created in 2015 (*Marušiak et al., 2015*). This version was based on the then current gravimetric database of Slovakia, compiled within the project APVV-0194-10, entitled: "Bouguer anomalies of new generation and the gravimetrical model of Western Carpathians" (*Pašteka et al., 2014*). Within this project, the existing regional gravity database (more than 212 000 points, which represented approximately 3–6 points per km^2) was supplemented with more than 107 000 detailed gravity points. Both parts of the database, regional and detailed, represent the results of several decades of gravimetric mapping and geophysical exploration in Slovakia (Fig. 1). CBA was calculated according to Eq. (1):

$$\mathrm{CBA} = \mathrm{g(P)} - \gamma(\mathrm{P_0}) + \delta \mathrm{g_F(P)} - \delta \mathrm{g_{sph}(P)} + \mathrm{TC(P)} + \delta \mathrm{g_{atm}(P)}\,, \qquad (1)$$

where g(P) is the measured gravity acceleration, $\gamma(\mathrm{P_0})$ is the normal gravity field (Pizetti-Somigliana formula with GRS80 reference system parameters) on the ellipsoid, $\delta \mathrm{g_F(P)}$ is the height (free air) correction term in a second degree approximation (*Wenzel, 1985*), $\delta \mathrm{g_{sph}(P)}$ is the gravitational effect of truncated spherical layer (*Mikuška et al., 2006*) with the truncation angle of

$1^\circ\,29'\,58''$ (corresponding to 166 730 m) and the density 2.67 g/cm^3, TC(P) is the terrain correction calculated to 166 730 m (2.67 g/cm^3), $\delta g_{\rm atm}$(P) is the atmospheric correction calculated by the effect of the true atmosphere (*Mikuška et al., 2008*), using the real topography model and the effect of spherical shell with radially dependent density (*Karcol, 2011*).

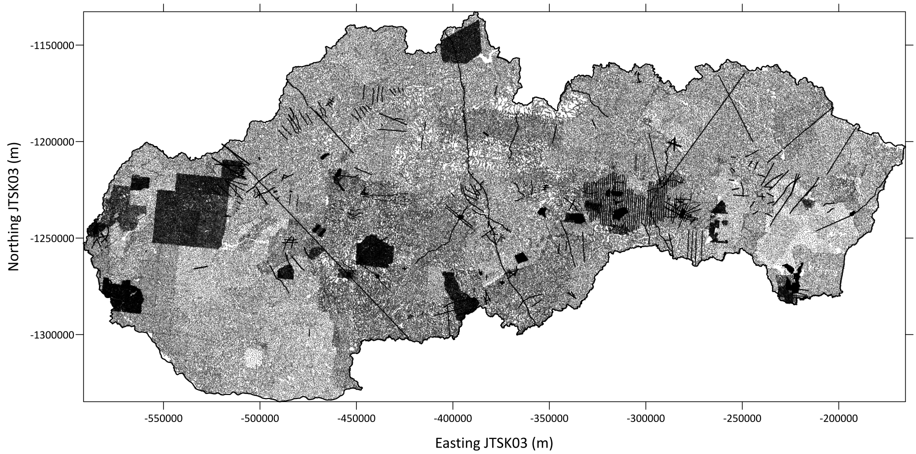

Fig. 1. Compiled gravimetric database of Slovakia in 2015 containing almost 320 000 gravimetric measurements (*Zahorec et al., 2017b*).

The calculation of terrain corrections in the four zones T1 (0–250 m), T2 (250–5240 m), T31 (5240–28800 m) and T32 (28800–166730 m) respects long-standing practice (*Grand et al., 2001; Zahorec et al., 2017a*), while in the nearest zone the most detailed DEM available at a given time is used. In the current version of the software, for Slovakia and its immediate surroundings we use a combined model, which was created by combining available local national digital terrain models based on laser scanning with an original resolution of 1 m (Slovakia, Czech Republic, Poland, Austria) with the global FABDEM model (*Hawker et al., 2022; Neal and Hawker, 2023*) with an original resolution of 1 arcsecond ($\sim$30 m) for Hungary, Romania and Ukraine. For Slovakia, the DMR 5.0 model was used with the original resolution of 1 m in the ETRS89-TM34 coordinate system in the original ellipsoidal heights in the ETRS89 system above the GRS80 ellipsoid (`https://data.slovensko.sk/datasety/cdfebbf0-d324-43fb-abc2-e04505b293f0`). The following models were used for the surrounding countries:

Poland – NMT-PL-KRON86-NH (https://www.geoportal.gov.pl/en/data/digital-elevation-model-dem/),
Czech Republic – DMR 5g (https://geoportal.cuzk.cz/),
Austria – DGM ALS DTM (https://data.bev.gv.at/).

Since DMR 5.0, as the primary part of the resulting model, was generated from highly accurate point clouds, the height accuracy of DMR 5.0 obtained based on control measurements on paved surfaces reaches values from 0.02 to 0.16 m. Height accuracy outside paved areas is 2–3 times worse, especially in dense vegetation (https://egako.eu/wp-content/uploads/2023/12/leitmannova_galova_2023_12.pdf).

3. CBA2G_SK software

The CBA2G_SK software was developed to calculate gravity at points of levelling networks in Slovakia. The algorithm consistently follows the back-calculation from interpolated values from the CBA map according to Eq. (2) (explanation of symbols is below Eq. (1)):

$$g(P) = CBA_{int} + \gamma(P_0) - \delta g_F(P) + \delta g_{sph}(P) - TC(P) - \delta g_{atm}(P)\,. \tag{2}$$

The same DEMs and the same division into calculation zones are used as in the original CBA map calculation. The user interface is shown in Fig. 2. In addition to the input data (calculation points and CBA grid), it contains four sections with settings and inputs for calculating each terrain correction zone. The program accepts input grid formats (CBA and DEMs) Surfer 6 bin, or Geosoft DOS. The limitation of the program for the territory of Slovakia is given by the use of the S-JTSK (JTSK03) and ETRS89 coordinate systems and conversions between these systems.

3.1. New version CBA2G_SK 2025

In 2025, the input data for the CBA2G_SK software was updated. Over the past 10 years, the database of gravimetric points has been expanded by several thousand points. In addition, thanks to international cooperation on the AlpArray project (*Zahorec et al., 2021*), we are able to supplement the map with border areas. So the new CBA map extends about 5 km beyond the borders of Slovakia (Fig. 3). Terrain corrections in nearby zones are calculated using the latest LiDAR-based DEMs (Chapter 2). However, in

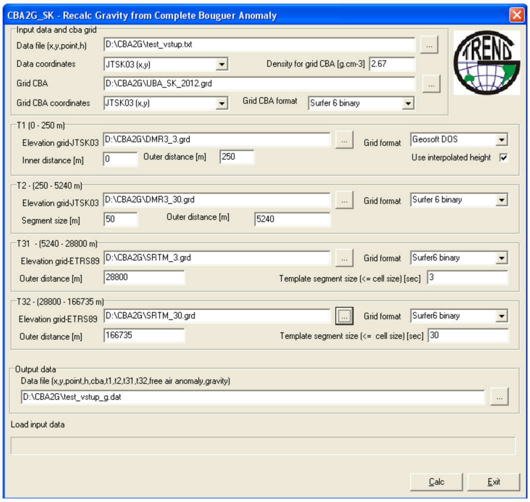

Fig. 2. User interface of the CBA2G_SK software.

this calculation, we must take into account the fact that the current DEM may not reflect the state of the topography during the period of gravimetric measurements (the last almost half a century). Moreover, in addition to real

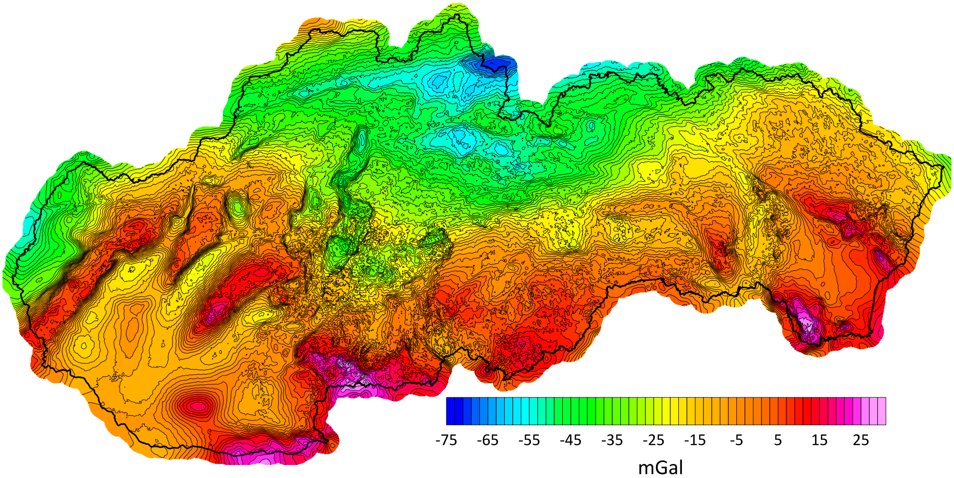

Fig. 3. New version of the CBA map of Slovakia and the surrounding area from 2025 for the needs of the CBA2G_SK software. The concept of ellipsoidal heights was used in the calculation of the CBA.

changes in topography (construction of roads, water reservoirs, expansion of surface quarries, etc.), another problem is that many database points have inaccurate coordinates, as GNSS technology was not yet available at that time. For this reason, this older part of the database was recalculated using previous versions of the DEM. And only the newer part of the database, which contains points measured over the last 20 years using precise terrestrial techniques and GNSS, was recalculated with the current LiDAR DEM. When back-calculating gravity, the current DEM will be used consistently to achieve the most realistic gravity values on the current topography.

In the new software version, we have changed the concept of the heights used. The previous version of the CBA map was calculated in the "classical" way using normal (physical) heights, namely Bpv (Bpv 1957 realization). Since, in addition to Bpv, the newer EVRS height system (EVRF2019 realization) is currently valid in Slovakia, we switched to the concept of ellipsoidal heights in ETRS89 when calculating CBA (in accordance with, for example, *Zahorec et al., 2021*). The reason is so that the user of the CBA2G_SK software can avoid height transformations. This means that even when back-calculating gravity, the user uses ellipsoidal heights as input data, which is simpler and more unambiguous. When using the concept of ellipsoidal heights, it was necessary to transform the local national DEMs originally provided in physical heights into ellipsoidal ones using the relevant national geoid/quasi-geoid models. The following models were used for individual countries: Poland – PL-geoid-2011 (*Kadaj and Świętoń, 2016*), Czech Republic – CR-2005-v1005 (*Kostelecký et al., 2004*), Austria – Austrian Geoid 2008 (*Pail et al., 2008*), FABDEM – EGM2008 (`https://earth-info.nga.mil/index.php?dir=wgs84&action=wgs84#egm2008`).

After transformation to ellipsoidal heights, the individual digital models were combined into a single unit and resampled to a step of 5 m (for T1 and T2 zones) and 1 arcsec (for T31 zone) using a bilinear transformation implemented in the QGIS environment using the GDAL library.

4. Testing the accuracy of gravity calculation using software CBA2G_SK

To test the accuracy of determining gravity by the CBA2G_SK program, we used sets of existing independent gravity measurements (not used in the calculation of the CBA map) covering the entire territory of Slovakia, as well as local measurements carried out for this purpose. Figure 4 shows the results of comparing the measured gravity and the values generated by the CBA2G_SK program for a set of 1366 points. This set is made up of points of order 0 to 3 of the State Gravimetric Network, realization S-Gr95. The maximum differences are at the level of approximately ±2 mGal with a standard deviation of 0.29 mGal.

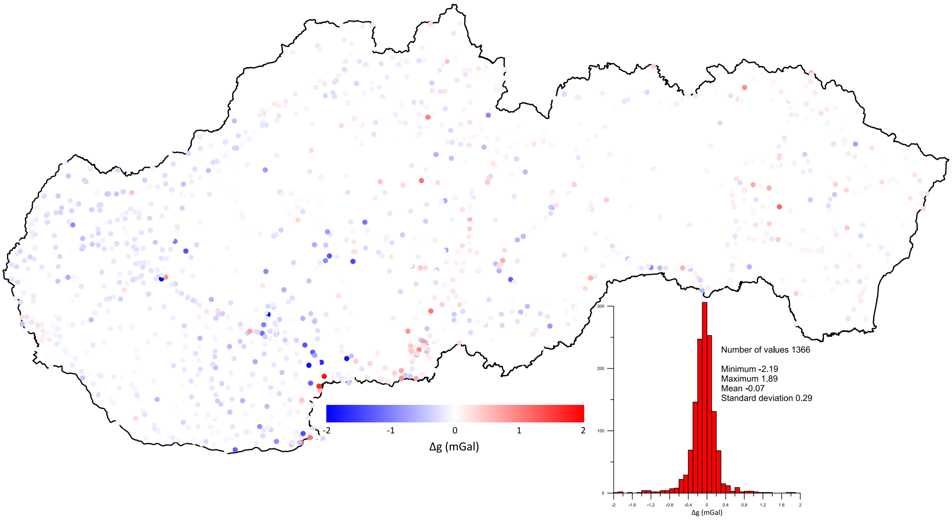

Fig. 4. Comparison of measured gravity values and those calculated by the CBA2G_SK approach on a set of independent measurements at points of the existing State Gravimetric Network.

Another set of independent measurements is represented by points of the new, upcoming State Gravimetric Network (Fig. 5). Modernizing the gravimetric network will ensure its more frequent updates, higher accuracy, uniform measurement methodology, and also lower maintenance costs (fewer points). The new gravimetric network consists of points determined only from absolute gravity measurements with an accuracy of up to ±5 μGal (1st order). Absolute gravity measurements were carried out between 2023 and 2025 using the FG5X instruments (STU Bratislava and VÚGTK Pecný) for the 0th order and A10 (IGIK Warsaw) for the 1st order (*Novák et al., 2025*). The obtained differences compared to CBA2G_SK at 86 points of the new absolute gravimetric network practically do not exceed ±1 mGal with

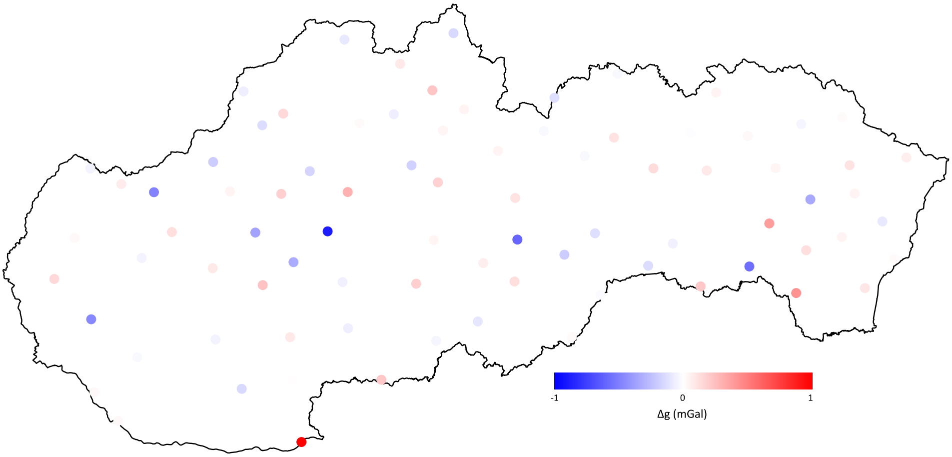

Fig. 5. Comparison of measured gravity values and those calculated by the CBA2G_SK approach on a set of independent absolute gravity measurements (prepared absolute gravimetric network).

a standard deviation of 0.25 mGal. The only exception is the southernmost point on the country's border (Fig. 5), where the difference reaches 1.3 mGal. The reason is probably the location of this point on the state border, where coverage by database points is limited.

4.1. In-situ verification

In addition to existing measurements, as part of testing the accuracy of the new version of CBA2G_SK, we focused on in-situ verification at locations where we can assume the largest errors in predicted gravity within Slovakia. We focused on two areas: the area of Central Slovakian Neovolcanics and the area of the Tatry Mts. (Fig. 6). The neovolcanic area is characterised by a pronounced "roughness" in the CBA map (Fig. 6), which is a consequence of the high variability of volcanic rock densities. It is likely that the shape of the CBA will be reflected in the interpolation error from the CBA map and thus in the increased error in the reconstructed gravity. Within the neovolcanites, we focused on the area of the Poľana caldera, which is also characterised by a relatively pronounced relief (the ridge altitude exceeds 1400 m above sea level). We measured 29 points of the levelling line from the centre of the caldera, across the ridge to the outer slopes of the caldera

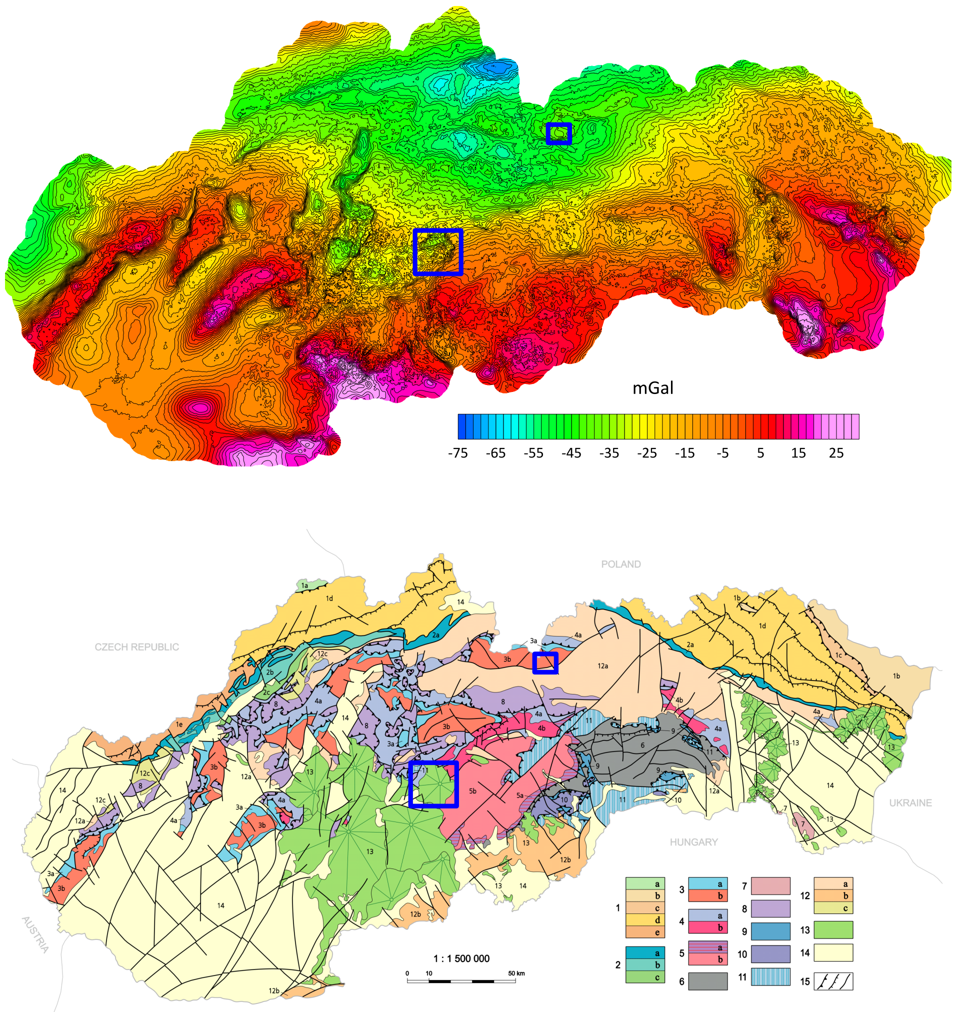

Fig. 6. CBA map (above) with areas marked for in-situ verification (blue rectangles). The bottom map shows the simplified geological map of Slovakia (modified after *Bezák et al., 2011*). Neo-Alpine tectonic units: 1 – Flysch Belt; 2 – Klippen Belt. Paleo-Alpine tectonic units: 3 – Tatricum; 4 – Fatricum and northern Veporicum; 5 – southern Veporicum; 6 – Gemericum; 7 – Zemplinicum; 8 – Hronicum; 9 – Meliaticum; 10 – Turnaicum; 11 – Silicicum. Formation superimposed over the nappe structure: 12 – Paleogene and Late Cretaceous sedimentary basins; 13 – Neogene and Quaternary volcanics; 14 – Neogene and Quaternary deposits.

(Fig. 7). As expected, the differences between in-situ measured gravity and predicted gravity are a little larger, at maximum they slightly exceed 2 mGal, the standard deviation is approximately 0.6 mGal.

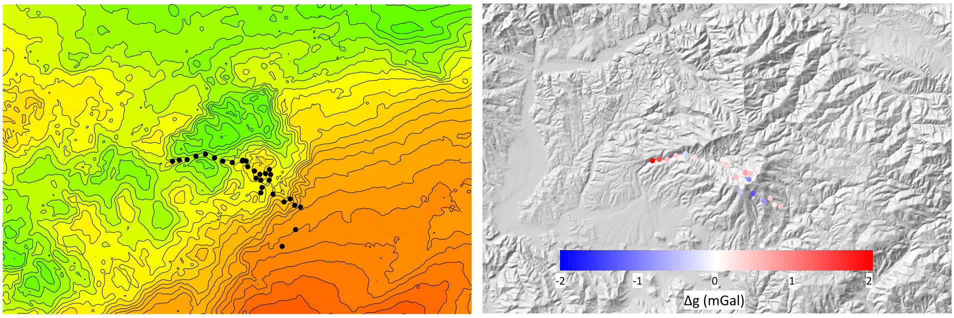

Fig. 7. Detail from the CBA map in the Poľana area (left) with marked in-situ verification measurements. On the right are shown the differences between the measured gravity values and those calculated by the CBA2G_SK approach (the shaded background shows the topographic relief of the caldera).

The second area we focused on in-situ verification of the CBA2G_SK prediction is the area of the highest peak within Slovakia – Gerlach peak. The reason we focused on this area is that the Tatry Mts. are the weakest covered by database points (Fig. 1), and therefore we can expect worse accuracy of gravity prediction. On the other hand, geologically this area is relatively monotonous, formed by granitoid rocks, which is also reflected

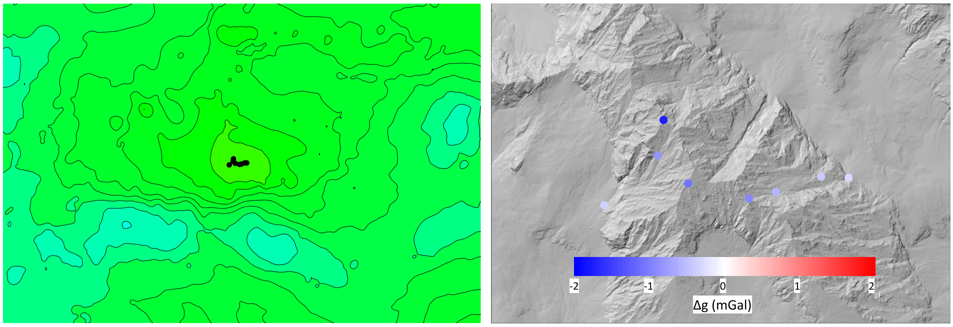

Fig. 8. Detail from the CBA map in the Gerlach peak area (left) with marked in-situ verification measurements. On the right are shown the differences between the measured gravity values and those calculated by the CBA2G_SK approach (the shaded background shows the topographic relief of the Gerlach massif).

in the CBA (Figs. 6 and 8). We conducted 8 in-situ measurements along the hiking route, including the highest point (2655 m.a.s.l.). The largest difference from the CBA2G_SK prediction, at the top of Gerlach peak, is close to 2 mGal (Fig. 8), the standard deviation is 0.5 mGal.

5. Discussion and conclusions

Based on the presented testing, we can conclude that the accuracy of gravity prediction using the CBA2G_SK software is on average ±0.3 mGal. The maximum expected differences from the measured values are approximately at the level of ±2 mGal, with 98 percent of the test points not exceeding an error of ±1 mGal. Height tests have verified that gravity determined by the CBA2G_SK software is very suitable for use in calculating normal heights in the State Levelling Network. The differences in heights, compared to using directly measured gravity, are at the level of tenths of a mm (*Majkráková et al., 2016*).

It is also interesting to compare the previous version of the CBA2G_SK approach (for normal heights) with the current version (for ellipsoidal heights). The statistical comparison (Table 1) is slightly in favour of the newer version, which is good news. Mutual differences in predicted gravity between individual versions do not exceed ±1 mGal. There is basically only one point that stands out from this statistic, where the difference between the older and newer versions is approximately 2.8 mGal. Upon closer analysis, we found that the given point is located in a relatively extreme position on the summit of Kľak (1352 m.a.s.l.) in Malá Fatra Mts. (Fig. 9). As we found out, the older version of the CBA2G_SK approach calculates the nearest terrain correction with a large error. The reason is the low-detailed DEM for the near zone used in the 2015 version (Fig. 9), which does not sufficiently capture the steep local topography, leading to an erroneous prediction. The difference between the measured value of gravity and the predicted values in the 2015 and 2025 versions is −2.5 and +0.3 mGal, respectively. There are no new measurements that would significantly update the CBA map in the given location. Therefore, this example illustrates the fact that the newer version of the DEM itself can locally significantly contribute to a more realistic prediction of gravity thanks to a more realistic calculation of the terrain correction in similar extreme situations.

Table 1. Statistical comparison of the differences between measured gravity values and those calculated by the CBA2G_SK approach for the 2015 and 2025 versions (in mGal).

Version	Minimum	Maximum	Mean	SD
2015	−2.47	1.89	−0.07	0.33
2025	−2.19	2.14	−0.07	0.31

So far, we have only discussed the gravity prediction on the Earth's surface, that is, where the measured data entering the calculation of the CBA map are. However, the CBA2G approach can also be generalized for the case of underground gravimetric measurements. In such a case, however, it would be necessary to introduce into the calculation the gravitational effect of underground objects such as tunnels (*Zahorec et al., 2023*) or caves (*Pašteka et al., 2024*).

Fig. 9. Comparison of the DEM used in the older version CBA2G_SK 2015 (left) and the new version 2025 (right) in the area of the Kľak summit in Malá Fatra Mts. The red cross indicates the gravimetric point.

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Data availability statement. The datasets generated during the presented study are available from the corresponding author on reasonable request.

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