

Higher-order regularised differentiations of potential fields

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Abstract: The differentiations of potential fields play an important role in their interpretations, as they are included in many of the enhanced transformations and edge detectors (Analytical Signal Amplitude, Tilt derivatives, Total Differentiation and others). However, the evaluation of horizontal and vertical differentiation is an unstable process. This instability was partially solved in the past by, e.g. Tikhonov regularisation for the first-order differentiations. Here, we present and compare several regularised stabilisations of higher-orders of differentiations. The methods are tested on synthetic and real data sets, where results indicate the usefulness of the presented method in the potential fields' processing and interpretation. Good properties of higher order regularised derivatives are demonstrated on selected edge-detection operator: in this case, on the modified balanced horizontal gradient operator.

Key words: Potential fields, Horizontal/Vertical differentiation, Tikhonov regularisation

1. Introduction

The instability of the first order differentiations of the potential fields is already well-known problem and several approaches were developed to mitigate this effect as: low-pass filtering of the input data in the spectral domain (*Butterworth, 1930*), beta-derivative concept (*Oliveira and Pham, 2022*), the input's upward continuation prior to the differentiation itself (e.g. *Ekinci et al., 2013; Oliveira and Pham, 2022*), or its stabilization via Tikhonov regularisation (e.g. *Cullum, 1979; Pašteka et al., 2009; Karcol and Pašteka, 2025*). This is of high importance, while the first order differentiations are the basis of many potential fields' transformation-operators (edge detectors as analytical signal amplitude (*Nabighian, 1972; Roest et al., 1992*). Another area of importance is the source position's estimators as Euler deconvolution (*Thompson, 1982; Reid et al., 1990*). The differentiations themselves

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are typically estimated in the spectral domain; however, the space domain solutions are available, too (finite difference schemes, e.g. *Oliveira and Pham, 2022*).

The resolution or efficiency of these interpretation methods could be improved by using the higher order differentiations (e.g. *Hsu et al., 1996; Abdelrahman and Abo-Ezz, 2001*), which brings the problem of their stabilisation into focus. The presented discussion is based on a close analysis of the first-order differentiations stabilised via Tikhonov regularisation as presented in *Karcol and Pašteka (2025)*.

2. Theory

The first option/idea how to obtain the higher order differentiation could be a “step-by-step” process, where the output of the first differentiation is the input for the second one and so on. Four different versions of the first order regularised differentiations were discussed in previous work (see *Karcol and Pašteka (2025)*, equations 16 – 18 and Table 1): a) The Classical form (CF), b) The Original form (OF), c) The General form (GF) and d) The Strakhov’s form (SF).

The n -times repeated first order differentiation of the input potential field U contaminated with error/noise δ , via these forms, gives us the following expression for the spectrum of the n^{th} order differentiation (tilde symbol denotes the entities in the spectral domain):

$$\tilde{f}_j^{(n)} = (\mathbf{D}_j \cdot \mathbf{C})^n \frac{\tilde{U}_\delta}{\prod_{k=1}^n [1 + \alpha_k S_f]}, \quad (1)$$

where $j = x, y, z$, α_k is the regularisation parameter (RP) for the k^{th} order differentiation, $\mathbf{C}$ is the vector of the classical differentiations in the spectral domain:

$$\mathbf{C} = (iu, iv, \sqrt{u^2 + v^2}), \quad (2)$$

$\mathbf{D}$ is the vector of the differentiation direction:

$$\begin{aligned} \mathbf{D}_x &= (1, 0, 0), \\ \mathbf{D}_y &= (0, 1, 0), \\ \mathbf{D}_z &= (0, 0, 1), \end{aligned} \quad (3)$$

and stabilisation term S_f (index f represents a type of form) is placed in Table 1. These versions are signed here by adding “S” to their names, e.g. GFS or SFS.

Table 1. The stabilization terms (*Karcol and Pašteka, 2025*).

Form f	S_f
CF (Classical Form)	0
OF (Original Form)	$u^2 + v^2$
GF (General Form)	$(u^2 + v^2) \mathbf{D}_j \cdot \mathbf{C}$
SF (Strakhov’s Form)	$(u^2 + v^2) \tanh(u^2 + v^2)$

The second option/idea is to obtain the regularised operator, which produces the higher order differentiation in a single step, since the step-by-step process could be time-consuming (the necessity of the search for several regularisation parameters) and could lead to the introduction of the regularisation error (no stabilisation/filtering is perfect). So, the goal is to find the modification of previously mentioned forms for the higher order differentiations:

- a) The CF acts no stabilization of the differentiation operator, so formulae for the n^{th} order differentiation are just n^{th} power of the CF operator, e.g. for the vertical differentiation we have:

$$\tilde{f}_z^{(n)}(u, v) = \left(\sqrt{u^2 + v^2}\right)^n \cdot \tilde{U}_\delta; \quad n \in \mathbb{N}. \tag{4}$$

Here, we can see that this form is no use for the solved problem since there is no damping of the higher frequencies’ amplification.

- b) The OF for the first order differentiation is described in *Pašteka et al. (2009)*. It is the result of the search for the minimum of the functional (via Euler-Lagrange equation), which describes the desired properties of the searched solution. The simple reconstruction of this process for the higher order differentiation gives the following expression (for the n^{th} order differentiation in the x direction):

$$\tilde{f}_x^{(n)} = (iu)^n \cdot \frac{1}{1 + \alpha(u^2 + v^2)} \cdot \tilde{U}_\delta, \tag{5}$$

where i is the complex unit. The first term of its spectral characteristic is the n^{th} order power of the spectral variable u (the CF in fact) and is

still “damped” by a quadratic function in the second term, only. Such damping is not enough, thus this approach is not useful for higher-order differentiations. To avoid this, we can ignore the functional altogether, and simply take the n^{th} power of the OF:

$$\tilde{f}_x^{(n)} = \left(iu \cdot \frac{1}{1 + \alpha(u^2 + v^2)} \right)^n \cdot \tilde{U}_\delta. \quad (6)$$

The formulae for the two other differentiations are created by analogy. The stabilisation term is now of higher power than the power in the numerator, so the damping could be enough.

- c) The formulation of the functionals which lead to the General Form introduced in *Karcol and Pašteka (2025)* can be modified for higher differentiations and yields:

$$\tilde{f}_x^{(n)} = (iu)^n \cdot \frac{1}{1 + \alpha(iu)^n(u^2 + v^2) \left[\operatorname{Re}\{(i)^n\} + |\operatorname{Im}\{(i)^n\}| \right]} \cdot \tilde{U}_\delta, \quad (7)$$

$$\tilde{f}_y^{(n)} = (iv)^n \cdot \frac{1}{1 + \alpha(iv)^n(u^2 + v^2) \left[\operatorname{Re}\{(i)^n\} + |\operatorname{Im}\{(i)^n\}| \right]} \cdot \tilde{U}_\delta, \quad (8)$$

$$\tilde{f}_z^{(n)} = (u^2 + v^2)^{\frac{n}{2}} \cdot \frac{1}{1 + \alpha(u^2 + v^2)^{\frac{n}{2}}(u^2 + v^2)} \cdot \tilde{U}_\delta, \quad (9)$$

where the term $\operatorname{Re}\{(i)^n\} + |\operatorname{Im}\{(i)^n\}| = \{-1; 0; 1\} \ \forall n \in \mathbb{N}$ avoids the possible singularity in the equations (7) and (8).

- d) The Strakhov’s Form faces a similar problem as OF – the way of its derivation is not known, so to adjust it for the higher differentiations, we simply put it to the n^{th} power (just like the OF) and this yields:

$$\tilde{f}_x^{(n)} = \left(iu \cdot \frac{1}{1 + \alpha(u^2 + v^2) \tanh^p(u^2 + v^2)} \right)^n \cdot \tilde{U}_\delta, \quad (10)$$

where $p \in \mathbb{N}$, however, $p = 1$ is used in this study (as well as in the *Karcol and Pašteka (2025)*). The formulae for the two other differentiations are created by analogy.

These versions (equations (6)–(10)) are signed here by adding “N” to their names, e.g. GFN or SFN.

3. Testing

The behaviour of the presented versions of the higher order regularised differentiation is tested on the synthetic data and one real data set (transformation of a selected complete Bouguer anomaly grid). The usefulness of the discussed versions is not dependent on the quality of the results only. We focus on the search for the RP as well. Among several options, we are using the C norm approach (*Tikhonov and Glasko, 1965*) for the setting of the quasi-optimal value of RP, which is usually searched in the local minima (or close in their vicinity). This led us to the discussion about the shape of the C norms for these versions as well. For more detailed insight into C norm usage, see e.g. *Karcol and Pašteka (2025)*.

Synthetic test

We use the model from *Liu et al. (2025)*, made of 3 rectangular prisms (see Table 2 for the model parameters) with a grid step of 0.25 km. The input field is corrupted by 5% truncated Gaussian noise – it makes noise values to be taken from interval $(-6.7; 6.7)$ mGal, which can be considered as strong noise for the gravity data (Fig. 1).

Table 2. Parameters of the model's prisms *Liu et al. (2025)*.

Body	Length [km]	Width [km]	Height [km]	Depth [km]	Density [g·cm ⁻³]
M ₁	16	16	2	6	2.5
M ₂	9	10	3	2	1.8
M ₃	5	5	1.5	2	1

Liu et al. (2025) used a deep learning technique to find vertical differentiations up to the 3rd order, and so, we are testing the presented forms for this case as well.

The C norms (normalised to number 1, logarithmic scale) for the vertical differentiations up to 3rd order obtained via the discussed versions of the differentiation operator are displayed in Fig. 2.

The GF (GFS and GFN) is the only version where the local minimum of the C norm exists for each differentiation order. On the other hand, there is a common situation when more local minima are present. So, there is a set of RP values to be tested – it is not necessarily the RP for the lowest

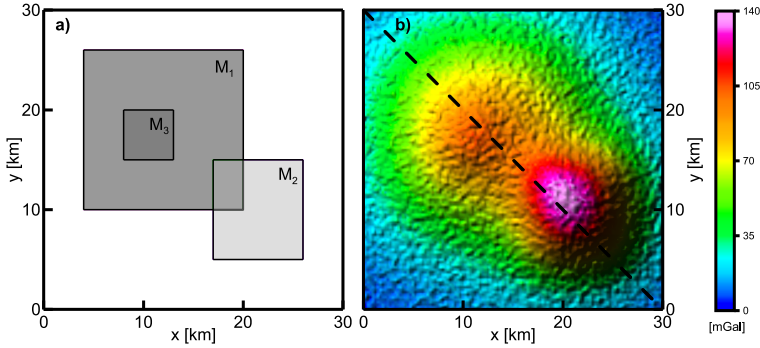


Fig. 1. The model for synthetic test: a) The positions of the model bodies, b) The input field (vertical component of gravitational attraction vector) corrupted by 5% truncated Gaussian noise with highlighted position of the control profile (black dashed line).

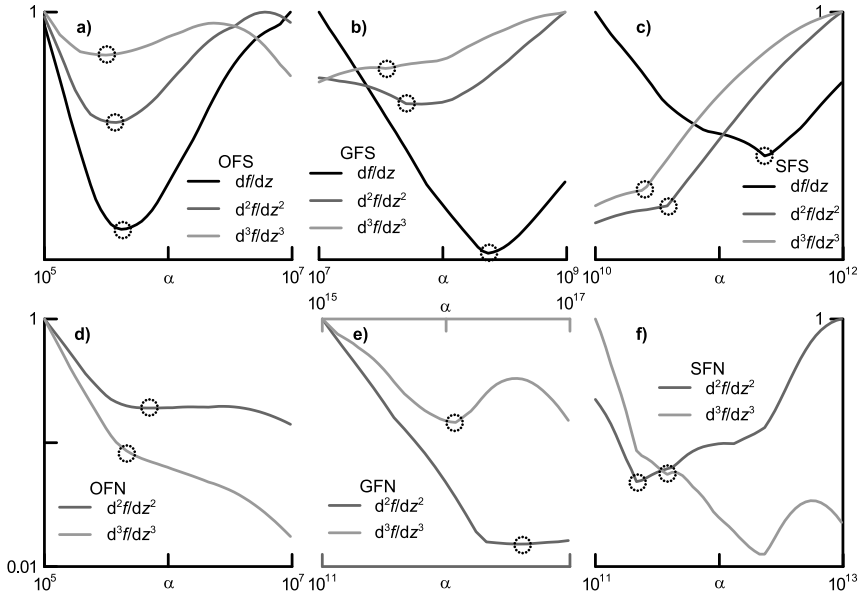


Fig. 2. The C norms for the discussed versions of the regularised operators for the vertical differentiation up to 3rd order of the input field f : a) OFS, b) GFS, c) SFS, d) OFN, e) GFN, f) SFN. The dotted circles highlight the positions of the used values of the regularisation parameter α .

of the C norm’s minima (which could be a tempting choice), see e.g. Fig. 2f for the third order differentiation via SFN. If there is no local minimum (e.g. OFN), we choose some “irregularity” (or “non-smoothness” points) of

the C norm curve as the final selection of the RP. The solutions obtained for the selected RPs are compared with the analytical solution on the control profile (Fig. 3 for “S” versions and Fig. 4 for the “N” versions of the regularised operators).

The solutions obtained by the “S” versions demonstrate the usability of this approach in combination with the C norm. However, the solutions for the OFS are a bit weaker – the fit to the analytical solution is quite good for the 1st and 2nd order differentiations. The OFS for the 3rd order gave

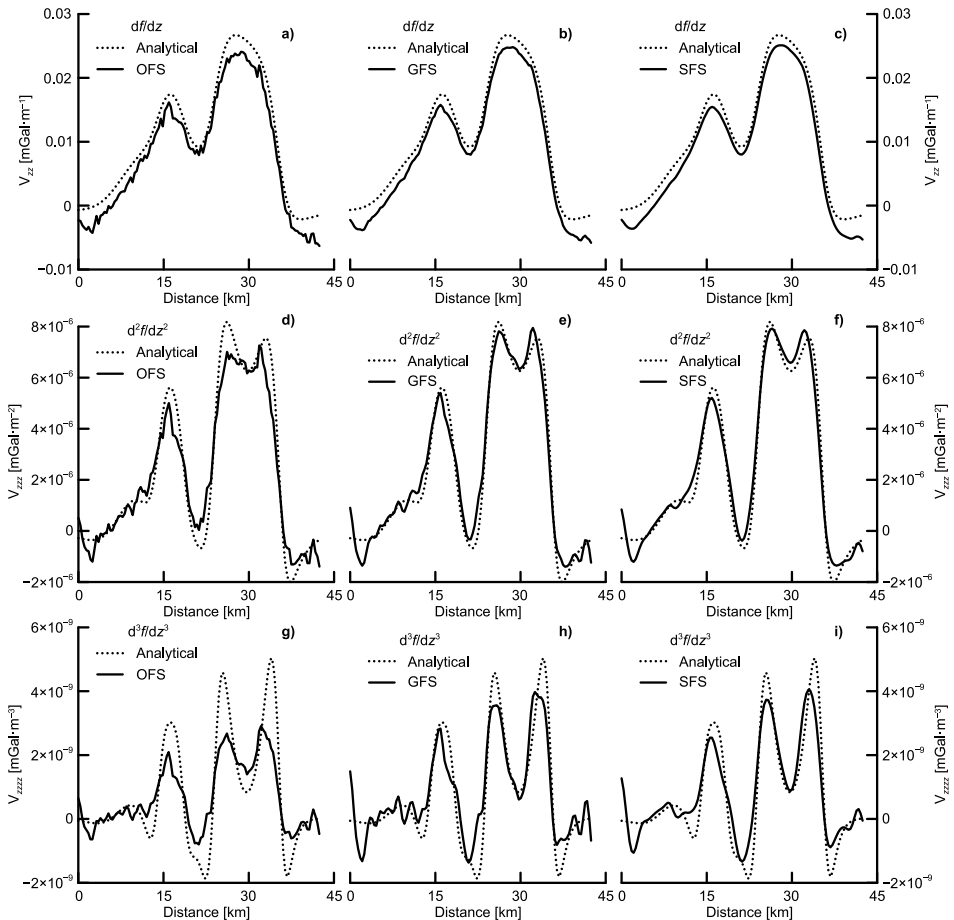


Fig. 3. The solutions for the vertical differentiation obtained via “S” forms on the control profile for the RPs selected by C norm (see Fig. 2).

a too-disturbed solution with a weak fit with the analytical solution. Two other versions (GFS and SFS) gave sufficiently good solutions for each differentiation order. This led us to the role of the C norm – some of the local minima gave acceptable solutions, some solutions are disturbed, sometimes the local minimum of the C norm does not exist. So, the local minimum is not necessarily the same as the C norm, and is not the only option for selecting final values of the RP. However, for this particular test of the “S” versions, the GFS gives better solutions and its C norm gave a local minimum for each differentiation order.

On the other hand, some of the “N” versions demonstrate the imperfection of the RP’s estimation C norm in this particular test – the 2nd order differentiations are in good quality, but the solutions of 3rd order are clearly over-regularised (too smooth) for GFN and SFN, and it is weak for the OFN. Despite the fact that the solution via OFN is not too smooth, the fit with the analytical solution is weak, so even if the RP is higher to obtain a smoother solution, the fit becomes worse. Here, the usage of the C norm’s minimum is clearly not successful in the estimation of the proper value of the RP,

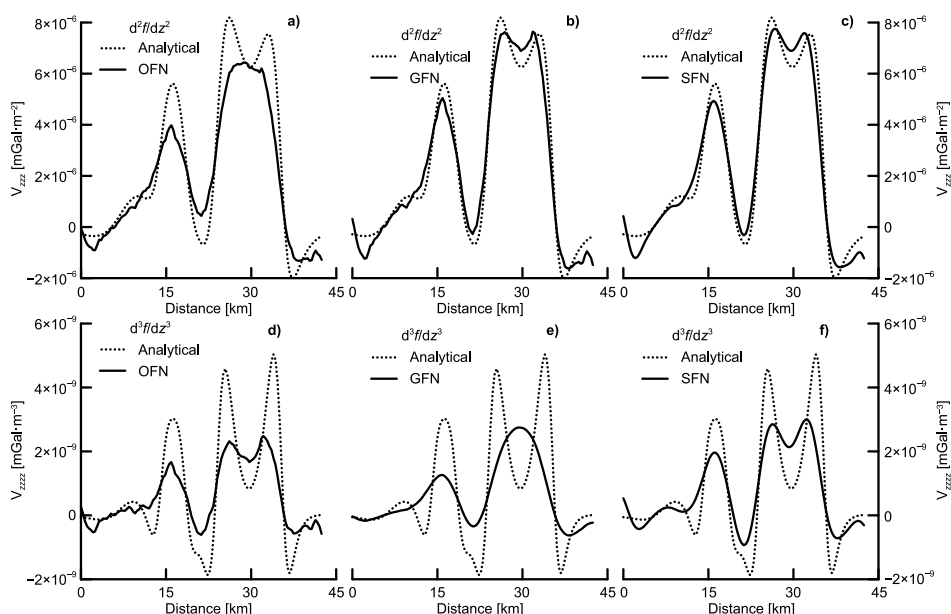


Fig. 4. The solutions for the vertical differentiation obtained via “N” forms on the control profile for the RPs selected by C norm (see Fig. 2).

so other methods have to be employed – with the trial and error method, the solutions for the 3rd differentiation order via GFN and SFN are getting better, see Fig. 5.

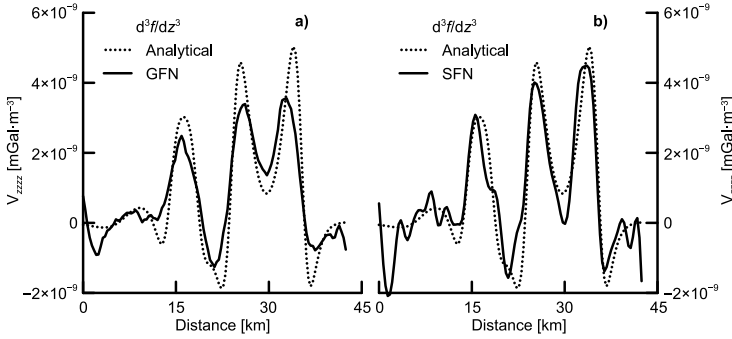


Fig. 5. The solutions (on the control profile) obtained with the trial and error method of the RP selection.

However, this means that it is still worth obtaining a minimum of the C norm – if not as the final selection of the RP, then as a good starting point for trial and error (or another) method.

The comparison of these results with those presented in the *Liu et al. (2025, Fig. 6)* shows that GF and SF (both versions) are less smooth, but the fit in the extremal values is better (for the 1st and 2nd differentiation, the comparison for the 3rd order is not possible because of the improper scale of the Fig. 6c scale of *Liu et al. (2025)*) with less computational effort time consumption.

The usage of the presented higher order regularised differentiations can be demonstrated on the edge-detection operators, e.g. the *TDXna* operator, which combines all three directions of differentiations and determines edges of anomalous objects by its zero value (*Fairhead and Williams, 2006*):

$$TDXna = \arctan \frac{\sqrt{\left(\frac{\partial U_\delta}{\partial x}\right)^2 + \left(\frac{\partial U_\delta}{\partial y}\right)^2}}{\frac{\partial U_\delta}{\partial z}}. \quad (11)$$

Note that there is a very similar formula of *Cooper and Cowan (2006)* available with the absolute value of the denominator. We add the “na” (“no absolute”) to the operator’s name to distinguish between these versions.

We modify this formula, as an experiment, by replacing the 1st order differentiation with higher orders to the following form:

$$TDXna^{(n)} = \arctan \frac{\sqrt{\left(\frac{\partial^n U_\delta}{\partial x^n}\right)^2 + \left(\frac{\partial^n U_\delta}{\partial y^n}\right)^2}}{\frac{\partial^n U_\delta}{\partial z^n}}. \quad (12)$$

While it is a zeroth isoline, what is interpreted when the $TDXna$ operator is applied, the evolution of this isoline (calculated analytically) with increased differentiation order is displayed in Fig. 6a, b, c. The shape of the discussed isoline is complicated while the model is made of overlapping bodies (dashed grey lines), but these analytical solutions show that higher-order differentiations could be useful in this particular usage. The results obtained for the noise corrupted data via GFN (the OFN and “S” versions are not used,

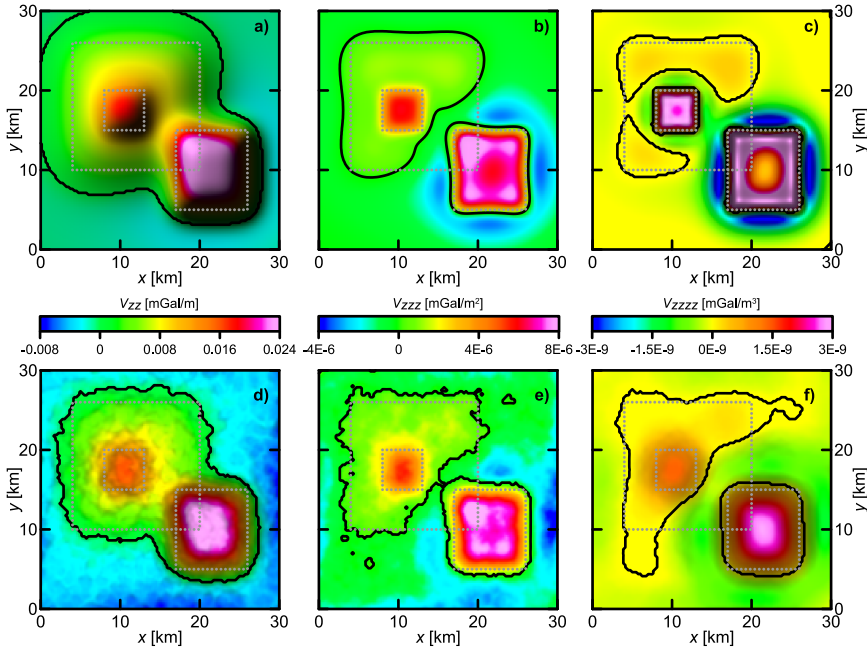


Fig. 6. The $TDXna^{(n)}$ results calculated analytically: a) $n = 1$, b) $n = 2$, c) $n = 3$ and results obtained via GFN: d) $n = 1$, e) $n = 2$, f) $n = 3$. The black lines highlighted the zeroth isoline of the $TDXna^{(n)}$ operator, placed over the analytical (a), b) and c)) or regularised (d), e) and f)) vertical differentiations of given order (coloured maps). The grey dotted rectangles stand for the edges of the model bodies.

and SFN looks similar) are displayed in Fig. 6d,e,f. All $TDXna^{(n)}$ results are placed over a map of the vertical differentiation of the given order for comparison.

These results show several important features. Firstly, the fit to the analytical solution is good enough for $n = 1, 2$ (Fig. 6d,e). The solution for the 3rd order differentiations (Fig. 6f) works well for the (almost) isolated body M_2 , but for the two overlapping bodies, only the larger and deeper body's boundaries were partially reconstructed (when compared to the analytical result). The reasons for this could be the too high noise level and the fact that the shape of the TDXna operator is governed mostly by vertical differentiation. These values are small (power 10^{-9}) and are more disturbed in the close vicinity of the zero value (compare with Fig. 4e and Fig. 5a). With smoother vertical differentiation, the resolution of the result

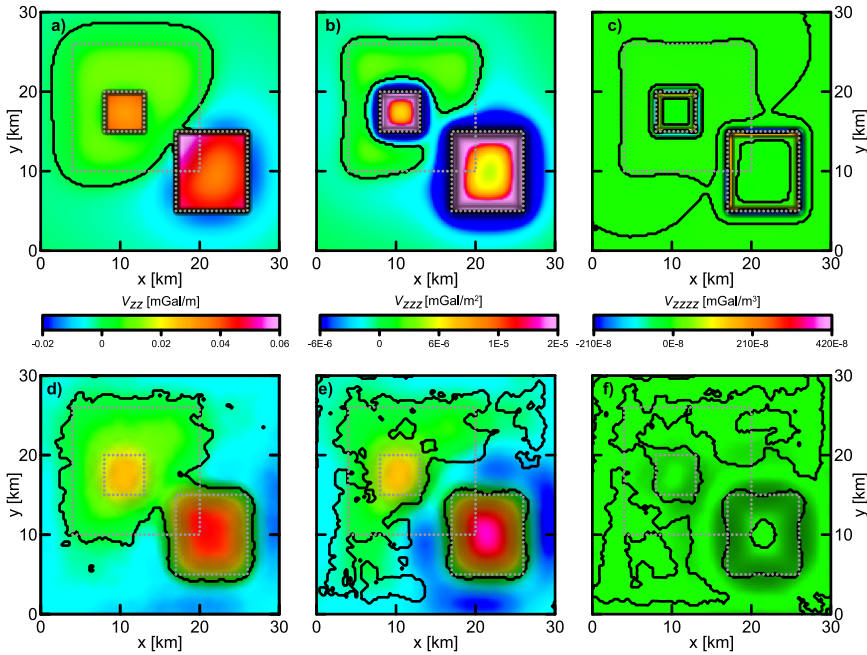


Fig. 7. The $TDXna^{(n)}$ results at DC level $h_{DC} = 1900$ m calculated analytically: a) $n = 1$, b) $n = 2$, c) $n = 3$ and results obtained via GFN: d) $n = 1$, e) $n = 2$, f) $n = 3$. The black lines highlighted the zeroth isoline of the $TDXna^{(n)}$ operator, placed over the analytical (a), b) and c)) or regularised (d), e) and f)) vertical differentiations of given order (coloured maps). The grey dotted rectangles stand for edges of the model bodies.

decreases significantly. A possible way to partially overcome these difficulties with resolution is to incorporate a regularised downward continuation (DC) operator, as described in *Karcol and Pašteka (2020)*, where the combined operator for the 1st order differentiation (taken in the General Form, while such an operator's combination is not derived for other forms) and DC. This combination can be enlarged for n^{th} order differentiation. The solutions for the DC depth $h_{DC} = 1900$ m are displayed in Fig. 7 (again, vertical differentiations' maps are displayed, too).

Here, the reconstruction of the bodies' boundaries is good for the 1st order differentiation and poorer for the other two. However, the upper one of the overlapping bodies is recognised in the Fig. 7c (3rd order differentiation), despite the complication in the interpretation of this result.

Of course, this particular application of the higher differentiation would require deeper and detailed investigation, but the presented results indicate the tactics in such interpretation – several results have to be obtained, and the final conclusion should be based on the evolution of the given maps with increasing differentiation order and/or DC level.

Real data test – regional gravimetry

The input field is the complete Bouguer anomaly map (CBA) for the correction density $2670 \text{ kg}\cdot\text{m}^{-3}$ of the Danube basin, Slovakia (*Zahorec et al., 2017*), see Fig. 8. The selected CBA area belongs to the northern part of the Pannonian Basin (e.g. *Rybár et al., 2024*). In the central part of the study area, the so-called Sered' elevation manifests as a positive anomaly. This uplift structure is of interest from the perspective of geothermal exploration in the area, and obtaining information about its structural composition could be very interesting. For this reason, our study focused on qualitative interpretation of higher-order vertical derivatives of the CBA field.

Again, the previously discussed $TDXna^{(n)}$ (up to the 3rd order) operator is used. Each of the C norms provides several local minima, which were tested to be quasi-optimal values of the RP. The norms for some of the required differentiations are displayed in Fig. 9 as an example, and the results are displayed in Fig. 10.

The resulting $TDXna^{(n)}$ fields (Fig. 10) shows the gradual breakdown of the positive anomaly of the Sered' elevation as the order of the vertical derivative increases – this is best illustrated by the result for the $TDXna^{(2)}$

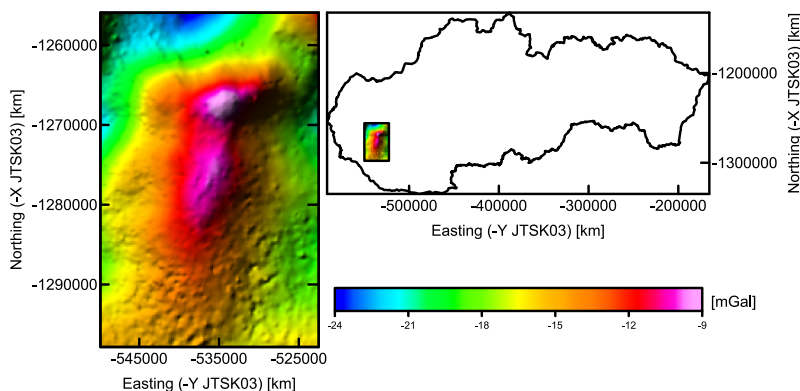


Fig. 8. The input field for the real data test – the complete Bouguer anomaly map of the Danube basin (correction density was $2670 \text{ kg}\cdot\text{m}^{-3}$) and its position within the borders of Slovakia (Zahorec *et al.*, 2017).

operator (Fig. 10b). The elevation is divided into its northern part (with a more pronounced amplitude of the first and second vertical derivative) and its southern part (with a lower amplitude). In the $TDXna^{(3)}$ field (Fig. 10c), the northern and southern parts even become completely separated from one another. Of particular interest is the presence of local depressions on the western side of the elevation, of which the central one is the most pronounced (with a Northing coordinate of its centre on the level of approximately $-1,280,000 \text{ m}$). The northern edge of this central local depression is also captured in the MXS2-93 reflection seismic profile (Fig. 11; the profile's location is shown in Fig. 10 as a white dashed line) – a hint

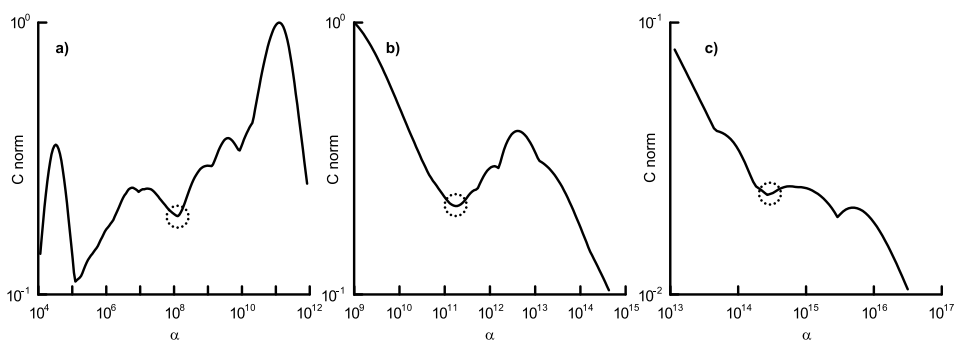


Fig. 9. The examples of the C norms (calculated for the GF) for the real data U for various directions and orders: a) $\partial U_\delta / \partial x$, b) $\partial^2 U_\delta / \partial y^2$, c) $\partial^3 U_\delta / \partial z^3$. The used values of the RP are highlighted by dotted circles.

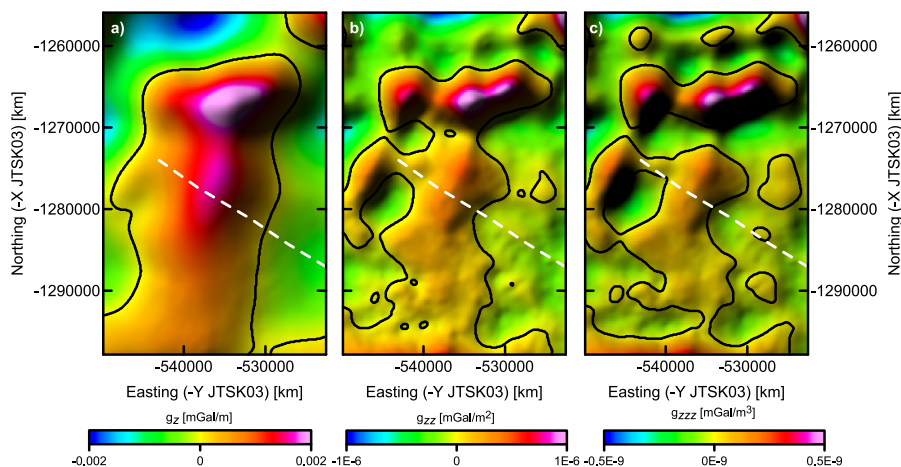


Fig. 10. The $TDXna^{(n)}$ results: a) $n = 1$, b) $n = 2$, c) $n = 3$, placed over regularised vertical differentiations' maps. The black lines highlighted the zeroth isoline of the $TDXna^{(n)}$ operator. The white dashed line shows the position of the seismic reflection line MXS2-93.

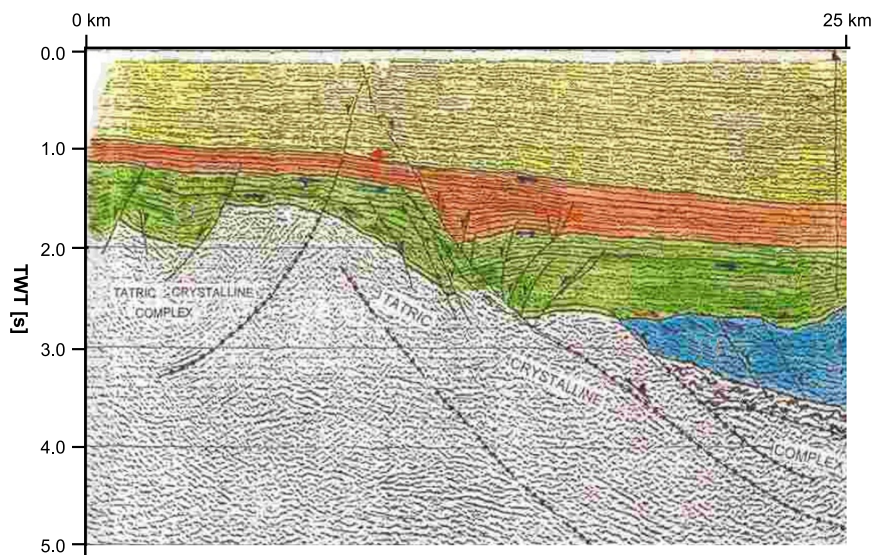


Fig. 11. Interpreted time section of seismic reflection profile MXS2-93 (Hrušecký, 1997). Coloured layers represent the sedimentary filling of the basin (yellow: middle Pannonian-Quaternary age, orange: lower Pannonian age, green and blue: lower Badenian-Sarmatian age). Pre-Tertiary basement (Tatric crystalline complexes) in the bottom part of the section is displayed without a colour fill.

of this depression is visible in the left part of the profile, corresponding to the course of the Pre-Tertiary basement within the studied elevation. The results obtained using the $TDXna^{(n)}$ operators may contribute to a better understanding of the tectonic structure of this studied formation in the future – despite the fact that the shape of the obtained $TDXna^{(n)}$ contour lines is of a rather atectonic nature.

4. Conclusion

Several expressions for the higher order regularised differentiations were presented as the logical next step from the first order cases discussed in the past. Several different shapes of two possible approaches of these operators were presented, tested and evaluated. The step-by-step process shows itself as a possible, but time-consuming approach with complications in the quasi-optimal value of the regularisation parameter estimation. Such a complication occurs for the single-step forms, but only single RP estimation is necessary, which makes these versions more suitable for practical applications (and lowers the regularisation error as well). However, the synthetic and real data tests show that possible usage of the 3rd (or higher) orders of differentiation is limited, due to their low amplitude and impossibility of completely removing the noise. As a good option, the calculation of all the differentiations of lower orders up to that, which is desired, is discussed. The evolution of the solutions with increasing of the differentiation order can be very helpful in the final interpretation of the results. Moreover, the possibility of the combination of the differentiation and downward continuation is briefly mentioned, too. The presented approaches and results clearly show the possibilities of the higher order differentiations in the potential field's transformations despite the still open question connected with the estimation of the quasi-optimal value of the regularisation parameter.

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