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A Model Reference Adaptive Controller based Flamingo Search Algorithm for Liquid Level Control in Non-Linear Conical Tank System

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Abstract: Conical tanks (CTs) play a vital role in the process industry as they can prevent residues in the tank. On the other hand, liquid level controlling in a CT is a complicated process due to the non-linear cross-sectional area. The complexity in CTs is also increased by the limitation to manipulated variables, interactions in measurements, and frequent disturbances. The control of liquid levels in CTs is of great importance in the process industry. In this paper, the model reference adaptive control (MRAC) with the modified proportional integral derivative (MPID) controller is proposed to maintain the liquid level at the set points. Two proportional integral derivative (PID) controller blocks are used in the proposed MRAC. The controller parameters are tuned using flamingo search optimization algorithm (FSA) based on minimum mean square error (*MSE*). The proposed method is implemented in MATLAB/Simulink platform and the results are considered with respect to different set points and disturbance conditions. Also, the results are compared with the existing controllers such as proportional integral (PI), PID, fractional-order proportional integral (FOPI) and MRAC-PID. The results show that the proposed MRAC provides lower integral square error (ISE) and integral absolute error (IAE) values of 38.25 and 2167, respectively.

Keywords: model reference adaptive controller, flamingo search algorithm, proportional integral derivative, process industry, conical tank, mean square error, liquid level

1. Introduction

The definition of a non-linear system is a system whose performance is not proportional to changes in the input. Due to its unpredictable changes, controlling a non-linear system is more challenging than controlling a linear system [1]. A few linear systems operate near the operating point in actual systems that are primarily non-linear in nature [2]. The process industry is characterized by non-linearity, which affects system performance. Due to variations in the tank's cross-sectional area, problems have developed in the non-linear system. Non-linear characteristics such as measurement delays, parameter uncertainties and interaction effects reduced the accuracy of industrial processes [3]. In practice, non-linear systems can be found in all process areas, and tuning such systems is a complicated task. Therefore, several controllers are developed in the process industry to overcome non-linearity [4], [5]. For linear tanks, such as cylindrical tanks, the design of controllers is generally a simple task. In contrast, modeling a process control system with minimal interaction is a challenging task in automatic control applications. Conversely, designing controllers for non-linear tanks is a tedious task in process control applications due to changes in cross-sectional areas [6].

Spherical tanks (STs) and conical tanks (CTs) are the two variants of the non-linear model where the limit of control becomes complex due to the shape of the tank. In a CT, the stored liquid can be completely emptied without leaving any residue [7]. In a real-time process, the problem arises in a non-linear system because in the event of an overflow, the liquid inflow is not proportional to the outflow rate [8]. In the process industry, maintaining the required liquid level is an important consideration, especially in CTs due to the non-linear structure. In such a system, the liquid level is the control variable performed by the percentage of valve opening. From an industrial point of view, liquid level control is a necessary task to make the system suitable for a variety

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of applications. Therefore, several controls have been developed in the past to cope with system variations. Most often, the proportional integral derivative (PID) controller is used in industrial applications because of its simplicity. However, the PID controller cannot provide better results when the system is subject to fluctuations and disturbances [9].

Initially, basic controllers such as proportional integral (PI) were used in industrial applications for liquid level control. Later, tuning approaches were introduced into the control architectures to improve the control behavior. On the other hand, sliding mode control (SMC) is used in the non-linear process control [10]. However, the processability of the PID controller is better in a linear system, but in the case of a non-linear system, it should be tuned with a minimum error. Therefore, metaheuristic algorithm based controllers are developed to provide notable solutions for non-linear applications. Here, the controlling complexity is added in the form of different cross-sectional areas [11]-[13]. In recent decades, fractional order based controllers have gained more attention in non-linear process control [14], [15]. The fuzzy logic approaches provide better performance under varying system conditions [16], [17]. An extensive comparison of current control approaches was made in [18], which shows that the existing PID controller leads to severe overshoot. Therefore, an efficient controller with lower peak overshoot is required for liquid level control. Several other controllers were placed in the CT with the goal of minimizing peak overshoot, but they used numerous optimization methods to fine-tune the parameters and increase performance.

From a controller tuning point of view, the model reference adaptive controller (MRAC) is the better choice. The MRAC provides the desired performance based on the reference model in non-linear process control [19], [20]. Therefore, the objective of the proposed work was to develop an MRAC-based controller for maintaining the liquid level in a conical water tank (CWT).

The contributions to this work are listed below:

1. Design an MRAC with a modified PID named D-PI controller for liquid level control under varying system conditions and disturbances.
2. Optimize the controller parameters based on the reduced *MSE* between the reference point and the liquid level of the tank.
3. Examine the performance of the proposed MRAC by implementing the existing approaches into the SIMULINK model of the CT.

The structure of the paper is as follows. Section 2 discusses related works, while Section 3 explains the methodological approach, such as the MRAC, the PID controller, and the flamingo search algorithm (FSA). The proposed method results are analyzed in Section 4, and the conclusion follows in Section 5.

2. RELATED WORKS

Recently developed controllers for liquid level control in the CWT are discussed in this section.

Arun and Sahaya Aarti [21] have developed a model of predictive control (MPC) for the CT. For linear models, a process model was adopted to calculate the output in the present state and the input in the future. In the gain scheduling MPC, the scheduler was used to tune the weights at the sampling moment and the weights were selected based on the manipulated inputs. Then, the set of weights is fed into to the plant.

Kumar et al. [22] proposed an internal model controller (IMC)-based PID controller for the CT to minimize the overshoot and settling time. This proposed method provided optimal controller parameters with respect to the variations in set points. The set point filter in this model was used for selecting the optimal tuning parameter to minimize overshoot. In this approach, an optimal tuning parameter was selected based on maximum sensitivity.

Rajiv Ranjan [23] developed MRAC for liquid level control in process industry. The proposed approach uses MIT rules for first and second order bounded disturbances. Unknown parameters of the plant were tuned using adaptive laws to produce zero tracking error.

Espitia-Cuchango et al. [24] developed a neuro-fuzzy adaptive control (NFAC) strategy for CT. In this proposed method, the parameters of the controller were optimized based on the identification of the plant in the system operation. In this approach, the descending gradient algorithm and back propagation algorithm were used to minimize the system error.

Aguila-Camacho et al. [25] proposed the fractional order PI for controlling the liquid level in the CT. In this method, the controller gains were tuned by the particle swarm optimization (PSO), which optimized the controller values based on 13 operating ranges. The results of the proposed model were verified under varying step changes. It was found that the PI controller has a higher steady state error than the proposed controller.

Balaska et al. [29] introduced a fractional-order model reference adaptive control (FO-MRAC) strategy for liquid level control in a CT system. The proposed FO-MRAC uses a fractional-order reference model to define the desired closed-loop performance characteristics. In addition, the control approach includes fractional integration in the parameter update process to improve adaptability. Both integer-order and fractional-order model reference adaptive controllers (FO-MRAC) are implemented for the non-linear system and their performance is compared to evaluate the effectiveness of the fractional-order approach.

Patil and Agashe [30] introduced a novel deep reinforcement learning (DRL) based control method to reduce the structural complexity and non-linear dynamics for liquid level control in CTs. The DRL controller regulates the liquid level in a CT by dynamically adjusting the inlet and outlet flow rates. The DRL agent interacts with the environment to optimize its control strategy and maximize cumulative rewards over time.

Ramanathan et al. [31] introduced machine learning-based controllers for the control of non-linear systems. In particular, a smart controller utilizing a reinforcement learning algorithm was proposed and evaluated for controlling the liquid level in a non-linear CT system. The system was modeled as a Markov decision process (MDP) and the control strategy was implemented using a reinforcement learning technique based on the Q-learning algorithm. The performance of the proposed controller was demonstrated by its ability to effectively regulate the liquid level in the non-linear tank system.

In this paper, a FO-MRAC strategy for controlling the liquid level in a CT system is presented. The proposed FO-MRAC uses a fractional-order reference model to define the desired closed-loop performance characteristics. In addition, the control approach includes fractional integration in the parameter update process for to improve adaptability. Both integer-order and fractional-order model reference adaptive controllers MRAC are implemented for the non-linear system and their performance is compared to evaluate the effectiveness of the fractional-order approach. Table 1 lists different methods for liquid level control in the CT.

CTs are used in a variety of industrial processes for drainage of slurries, viscous liquids, and solid mixtures. Level control in a CT is difficult due to its non-linearity and constantly changing cross-section. The main objective of this paper is to develop a suitable controller for the CT system to maintain the desired level. It is clear from the literature that an efficient controller is required to improve the time domain and time integral performance measures.

Table 1. Literature review.

Author	Method	Outcome	Advantages	Drawbacks
Arun and Sahaya Aarti [21]	MPC	Rise time is 262 s, overshoot is 9.3 %	Lower peak and settling time	Overshoot is higher in the second tank
Kumar et al. [22]	IMC-PID	ITAE is 0.0661, overshoot is $7.611 \cdot 10^{-4}$	It can be applied to the conical transfer function	Poor robustness
Rajiv Ranjan [23]	MRAC	Settling time is 30 s	System performance is stable under unmodelled dynamic	Longer settling time
Espitia-Cuchango et al. [24]	NFAC	Desired response is obtained under varying system conditions	Suitable for multi-input and multi-output systems	More settling time
Aguila-Camacho et al. [25]	FOPI	ITAE is $1.4465 \cdot 10^{6}$	Lower variance	More time to settle
Balaska et al. [29]	FO-MRAC	Sampling period is 0.1 s	Lowest error cost	Increases the noise
Patil and Agashe [30]	DRL	Desired response is obtained under varying system conditions	Reduces the complexity and non-linearity	Higher rise time
Ramanathan et al. [31]	Reinforcement learning algorithm	Settling time for trial 1 is 452 s	Reduces the non-linearity issues and settling time	Learning process slow

3. Proposed methodology

In liquid level control of a CT, the level of the liquid is kept within the desired level depending on the application. The controller used in these applications should minimize the difference between the liquid level in the tank and the reference level of the liquid. In addition, the controller parameters should be set to minimize overshoots. Taking these considerations into account, the MRAC-based controller is proposed in this work. The MRAC allows a plant to follow a reference model regardless of the variations of the plant parameters; therefore, it minimizes the difference between the reference and the output of the plant. In contrast, an adaptive controller without a stable tuning method causes instability in the system, and the improved PID controller is used together with the MRAC. First, a suitable model is developed using the system identification procedures, then the parameters are tuned using the FSA.

A. Mathematical modelling of CT

Conical shapes are still the strongest structure; they can withstand the greatest pressure. A low surface-to-volume ratio reduces the amount of heat entering the tank. High-pressure liquids are not a problem for the CT. Such tanks are used to store fuels, cryogenic liquids, and other materials in various industries. Fig. 1 shows the basic structure of a CT in which the liquid level is controlled at a steady rate by controlling the tank F input.

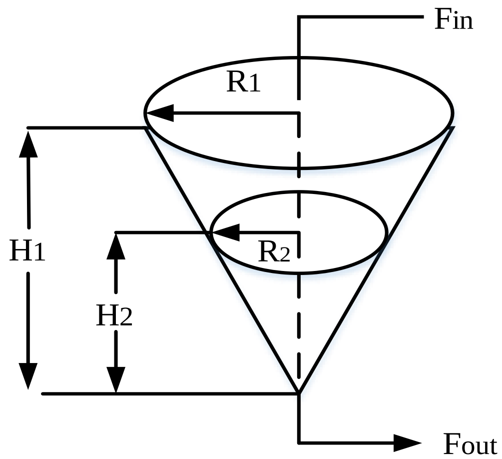

Fig. 1. Structure of a CT.

The accumulation rate of the CT A_{Rate} is analyzed using the following equation:

$$A_{\mathrm{Rate}} = F_{\mathrm{in}} - F_{\mathrm{out}} \tag{1}$$

where F_{in} is the flow rate of the incoming stream, F_{out} is the flow rate of the output stream.

$$\frac{\mathrm{d}V}{\mathrm{d}t} = F_{\mathrm{in}} - F_{\mathrm{out}} \tag{2}$$

$$F_{\text{out}} = \frac{H_2}{R_V} \tag{3}$$

where R_1 is the radius of the large CT, R_2 is the radius of the CT in the steady state, H_1 is the height of the CT, H_2 is the height of the CT in the steady state, and R_V is the resistance of the value. The volume of the tank V is analyzed by the following equation:

$$V = \frac{1}{3}\pi\, R_2^2\, H_2 \tag{4}$$

$$\frac{\mathrm{d}H_2}{\mathrm{d}t} = \frac{1}{\pi\, H_2^2 \lambda^2}\left(F_{\text{in}} - \frac{H_2}{R_V}\right) \tag{5}$$

$$\lambda = \frac{R_1}{H_1} = \frac{R_2}{H_2} \tag{6}$$

where λ is the ratio of radius to height of the CT [26].

B. Proposed MRAC

The goal of MRAC is to create a responsive control system that adjusts the controller's parameters so that the actual output of the plant matches the output of a reference model. MRAC techniques are used to control the level of the non-linear tank by variable manipulation of the tank's liquid level. An adjustment or adaptation mechanism and a reference model are integrated into the MRAC. The MRAC aims to design a closed loop controller with parameters that can be varied based on the system response. The system's output is then compared with the desired response of the reference model. The error difference between the plant output and the reference model output is used to adjust the controller settings. In addition, the controller input and the measured output are required for the investigation of the reference model. Fig. 2 shows the block diagram for the basic MRAC.

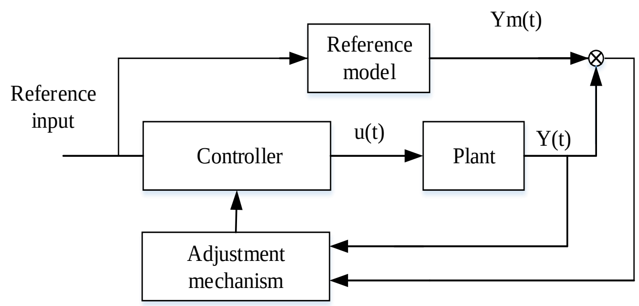

Fig. 2. Block diagram of MRAC.

The adaptive gain θ is multiplied by the controller's output to achieve the system's adaptability to changes.

$$U(t) = \theta * U_q(t) \tag{7}$$

where θ represents the adaptive gain in the MRAC structure, which adjust dynamically based on the system error to ensure the plant output follows the reference model. The term $U_q(t)$ represents the control signal generated by the adaptive MRAC with the D-PI structure. It is applied to the system to minimize the error between the reference model output and the plant output.

The D-PI controller is proposed in the MRAC to improve the control mechanism. The proposed D-PI controller is the improved version of the PID controller. The D-PI consists of two PID blocks in the first block, the proportional and integral gains are kept at zero. In the second block, the derivative gain is kept at zero.

$$U_{\text{PI}}(t) = K_{\text{p}} e(t) + K_i \int e(t)\mathrm{d}t \tag{8}$$

$$U_{\text{PID}}(t) = K_p e(t) + K_{\text{i}} \int e(t)\mathrm{d}t + K_{\text{d}} \frac{\mathrm{d}e(t)}{\mathrm{d}t} \tag{9}$$

Here K_{p}, K_{i}, and K_{d} are proportional, integral, and derivative gains. The error between the reference model and the actual output is measured using the following equation:

$$e\,(t) = Y(t) - Y_m(t) \tag{10}$$

where $Y(t)$ is the actual plant output and $Y_m(t)$ is the reference model output.

In the proposed MRAC, the controller parameters are modified by the FSA based on the minimum error. The adaptation algorithm modifies the controller parameters when the reference parameters are varied. The basic flow of the proposed MRAC is shown in Fig. 3.

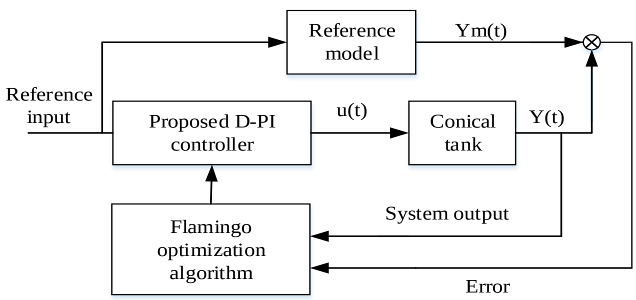

Fig. 3. Proposed MRAC with D-PI.

In this proposed work, the error generated from the reference model of the CT and the set points is passed to the FSA to tune the optimal controller parameters. The reference model of the CT is designed as the expected response of the plant model. The reference model requires a lot of experience with the dynamic system. In the case of liquid level control, an overshoot of 2 % and a settling time of 180 s are considered for the reference modeling [27], which is given by:

$$\gamma = \frac{\ln(P/100)}{-\pi}\sqrt{\frac{1}{1+\left(\frac{\ln(P/100)}{-\pi}\right)^2}} \tag{11}$$

Here γ is the damping ratio, the natural frequency $\omega = \frac{4}{\gamma t_s}$. The transfer function for the reference model is given as follows:

$$G(s) = \frac{3.55}{s^2 + 2.66\, s + 3.55} \tag{12}$$

FSA for parameter tuning

The FSA is a recently developed metaheuristic optimization method inspired by the foraging behavior and group dynamics of flamingos. This algorithm mimics their collaborative strategies for finding food-rich regions while balancing exploration and exploitation during optimization. [28]. A population of potential solutions (flamingos) is randomly initialized in the search space. Each flamingo represents a candidate set of controller parameters. Each flamingo evaluates its fitness based on an objective function. In this study, the mean square error (*MSE*) between the reference model and the actual liquid level of the CT is used as the fitness function. Based on this consideration, the fitness function of the FSA is formulated and expressed in the following equation:

$$\text{Fitness} = \\ = (w_1 \cdot MSE) + (w_2 \cdot P) + (w_3 \cdot T_{\text{settling}}) + (w_4 \cdot T_{\text{rise}}) \tag{13}$$

Here *MSE* is represented as mean square error, P is represented as peak overshoot, T_{settling} is represented as settling time, T_{rise} is represented as rise time, w is the weight of the objective function and varies between 0 and 20. The moving distance of n^{th} flamingo in the m^{th} dimension of the population at iteration *t* during foraging can be expressed as:

$$f_{nm}^t = \in_1 \cdot Xf_m^t + C_2 \cdot |\, C_1 \cdot Xf_m^t + \in_2 \cdot X_{nm}^t \,| \tag{14}$$

where f_{nm}^t represents the position of the n^{th} flamingo in the m^{th} dimension of the population, X_{nm}^t represents the position of the n^{th} flamingo in the m^{th} dimension of the population at iteration t, C_1 and C_2 are random numbers and $\in_1, \in_2$ are standard normal distributions. The position of the food-rich region in the m^{th} dimension is Xf_m.

The equation for modifying the position of the flamingo during foraging at iteration $(1+t)$ is:

$$X_{nm}^{1+t} = \frac{(X_{nm}^t + \in_1 \cdot Xf_m^t + C_2 \cdot |\, C_1 \cdot Xf_m^t + \in_2 \cdot X_{nm}^t \,|)}{G} \tag{15}$$

where G denotes the diffusion factor and the equation for the movement of the flamingo population is:

$$X_{nm}^{1+t} = \beta \cdot (Xf_m^t - X_{nm}^t) + X_{nm}^t \tag{16}$$

The location of the n^{th} flamingo in the m^{th} dimension of the population at iteration $(1+t)$ is represented by X_{nm}^{1+t} and β is a Gaussian random number. At iteration t, Xf_m^t denotes the m^{th} dimension location of the flamingo with the best fitness in the population.

The FSA procedure is given below:

- Step 1: Set the total flamingo population P, the maximum number of iterations $I_{\max}$ and the migration proportion PM_a.
- Step 2: In the n^{th} iteration of the flamingo population update, the population is divided into three groups based on the proportion parameter PM_a. The number of foraging flamingos is calculated as $P_R M = R[0,1] \cdot (1 - PM_a) \cdot P$, where $R[0,1]$ is a random number between 0 and 1, and P is the total population size. The number of flamingos in the first section is $P_{CM} = PM_a \cdot P$. The number of flamingos migrating in the second section is $P_{SM} = P - P_{RM} - P_{CM}$. Individual flamingo fitness values are determined and the population of flamingos is organized according to individual flamingo fitness values. The flamingos in the first section P_{CM} with low fitness and flamingos in the second section P_{SM} with high fitness are considered as migratory flamingos, while the remaining flamingos are considered as foraging flamingos.
- Step 3: Foraging flamingos update their position using (15) and migratory flamingos update their position using (16).
- Step 4: Ensure that each updated flamingo position remains within predefined bounds of the controller parameter space. If a flamingo exceeds these bounds, it is corrected either by limiting it to the nearest boundary value or by reinitializing it randomly within the valid range.
- Step 5: If the iteration count reaches $I_{\max}$, terminate and return the flamingo with the best fitness (optimal controller parameters). Otherwise, return to step 2 for the next iteration.

The FSA-tuned controller reduces the rise time and settling time compared to other optimization methods, resulting in faster system responses. FSA uses fewer parameters compared to techniques such as differential evolution (DE) or artificial bee colony (ABC), making it easier to implement while maintaining computational efficiency. Since FSA strikes a balance between exploration and exploitation, it converges to optimal solutions faster than traditional methods such as PSO, where more iterations may be required to fine-tune the solutions.

4. RESULTS AND DISCUSSION

The proposed work is carried out in MATLAB/ SIMULINK platform and the results are measured in different operational domains. The Simulink model of the proposed work is shown in Fig. 4.

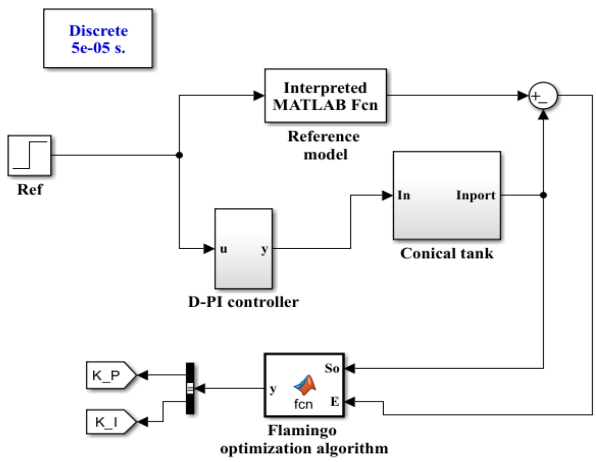

Fig. 4. Simulink model of MRAC with CT.

The time domain analysis of the different controllers is shown in Table 2. The results obtained with the proposed model are compared with the existing PI, PID, FOPI, and MRAC with PID controllers using the implemented models. First, the set point for the liquid level is set to 25 cm. The results are shown in Fig. 5. The model reference adaptive control-modified proportional integral derivative (MRAC-MPID) controller shows better performance in terms of lower overshoot, faster settling time, and faster rise time than the existing controllers. Specifically, the MRAC-MPID achieves an overshoot of 0.8 %, which is significantly lower than other controllers (e.g., FOPID: 2.8 %, MRAC: 1.6 %). This lower overshoot can be attributed to the precise tuning of the controller gains with the FSA, which optimizes the balance between stability and responsiveness. The settling time for MRAC-MPID (200 s) is also shorter compared to MRAC (245 s) and FOPID (273 s), demonstrating the improved adaptability and transient response provided by the MPID structure within the MRAC framework. The FSA ensures that the controller gains are tuned to minimize error metrics such as integral square error (ISE) and integral absolute error (IAE), resulting in improved steady-state accuracy and transient performance. For example, MRAC-MPID achieves an ISE of 38.25 and an IAE of 2167, both of which are lower than other controllers. Furthermore, in scenarios with varying set points or disturbances, MRAC-MPID shows faster recovery and lower deviation compared to MRAC alone. This is evident in the smooth transition curves and minimal oscillations shown in Fig. 5.

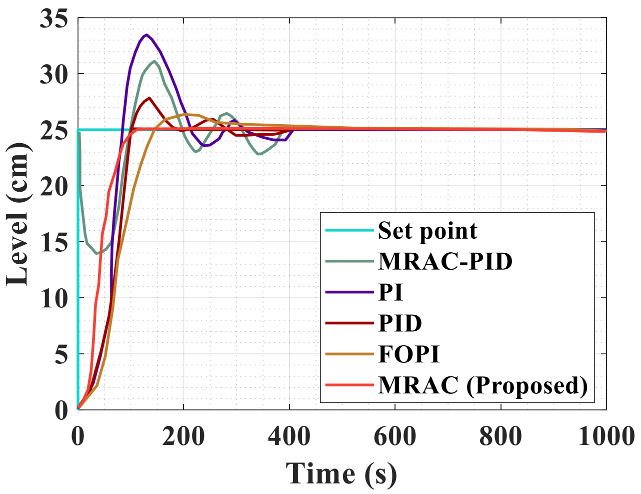

Fig. 5. Controller output under constant set point.

Table 2. Comparative analysis of different controllers.

Controllers	Peak overshoot [%]	Settling time [s]	Rise time [s]	ISE	IAE
PI	18.17	402	109	47.73	2681
PID	21.32	421	114	51.03	2968
FOPI	12.74	408	157	43.09	2902
MRAC-PID	7.19	391	174	41.35	2863
MRAC (Proposed)	0.8	200	103	38.25	2167

The performance analysis of controllers in response to random disturbances in the input flow is shown in Fig. 6. Here, the disturbance is applied at 500 s in the input flow and the results are measured for all controllers. The results are measured to evaluate the controllers' ability to reject disturbances and maintain the liquid level at the desired set point. The PID controller exhibits noticeable oscillations and delayed recovery after the disturbance. This is primarily due to its fixed gain parameters, which limit its ability to handle the non-linear dynamics and unpredictable changes in the system. The FOPID controller performs better than the traditional PID, but lags behind the MRAC-MPID in terms of disturbance rejection. The fractional-order terms provide some degree of adaptability, but are not sufficient to efficiently handle the randomness and magnitude of the disturbances. The standard MRAC with a PID controller shows better adaptability compared to PID and FOPID, but lacks the fine-tuned response of the MRAC-MPID. The lack of MPID leads to increased oscillation amplitudes and prolonged stability, which emphasizes the significance of the MPID enhancement. The MRAC-MPID demonstrates superior performance by tracking the set point with minimal oscillations after the disturbance is introduced. The adaptive nature of the MRAC ensures that the controller dynamically adjusts to the non-linear characteristics of the CT, while the MPID component improves damping, reducing the amplitude of the oscillations.

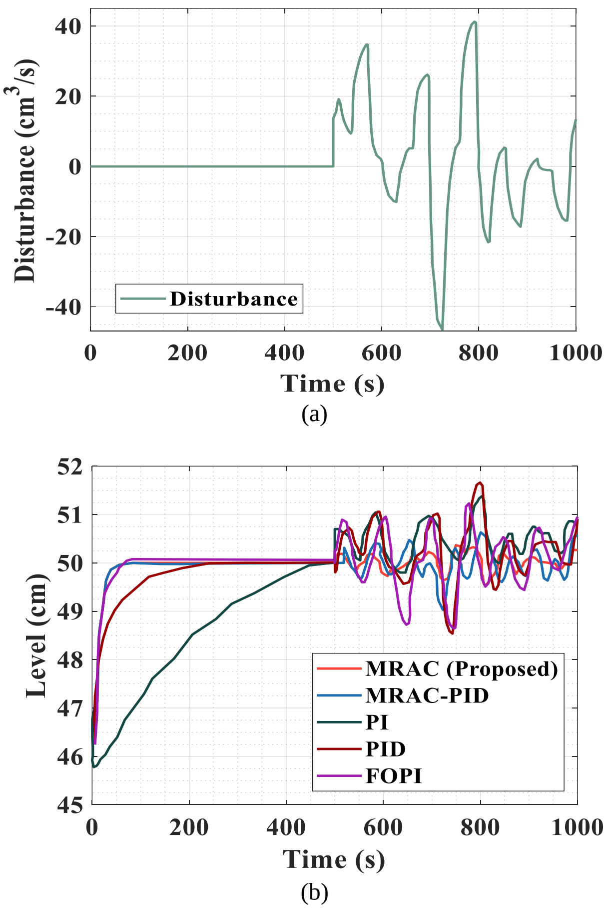

Fig. 6. Performance analysis under random disturbance in constant setpoint.

Controller performance under sudden disturbance in the load is shown in Fig. 7. In this case, the disturbance in the liquid level occurs between 300 s to 700 s. The PID controller struggles to cope with the abrupt load changes, resulting in significant deviations from the reference level. The FOPID controller performs better than the PID due to its fractional-order terms, which allow for some flexibility in response. However, it still cannot match the precision and adaptability of MRAC-MPID, as evidenced by the larger oscillations and slower recovery times. While the standard MRAC-PID controller has better adaptability than PID and FOPID, it lacks the fine-tuned control offered by the MPID enhancement. The MRAC-MPID controller handles sudden changes in load with minimal impact on the liquid level, ensuring process stability. The combined benefits of MRAC's adaptability and MPID's improved control precision minimize the effect of load disturbances, resulting in negligible overshoot and rapid stabilization.

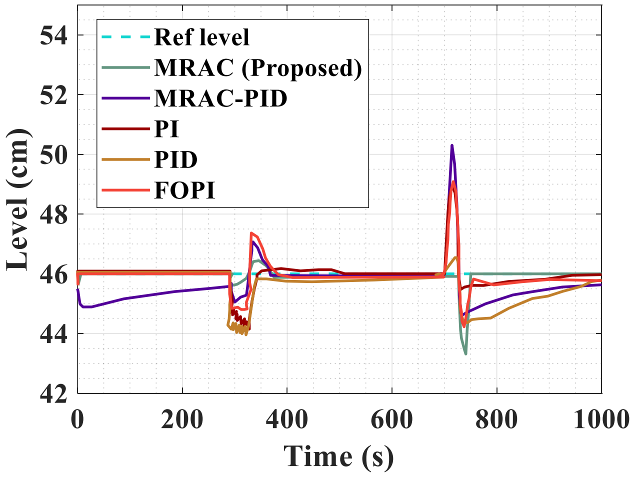

Fig. 7. Performance analysis under load variations.

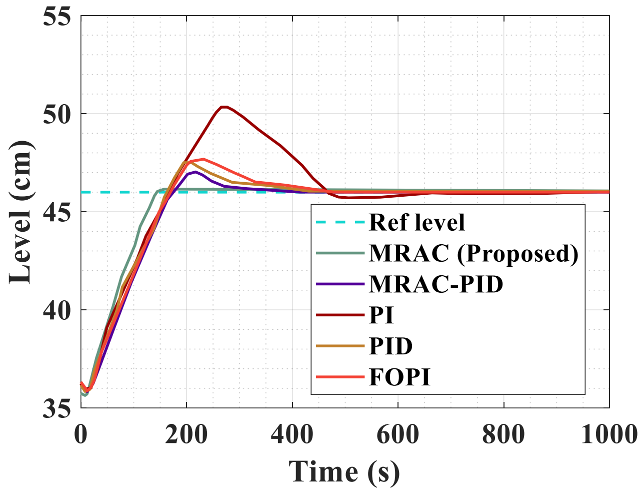

Fig. 8. Controller output under variation in reference from 30 cm to 46 cm.

The results of the controllers when the liquid level varies between 30 cm and 46 cm are shown in Fig. 8. The PID controller has difficulty dealing with the non-linear characteristics of the CT system, resulting in greater overshoot and prolonged settling time. The FOPID controller offers better adaptability compared to the PID controller due to its fractional-order terms. As a result, the FOPID controller has a higher overshoot and a slower response time during input range transitions. The MRAC-MPID controller achieves better liquid level regulation during input range variations, with minimal overshoot and smooth transitions compared to the existing techniques. The controller quickly stabilizes the liquid level after a change in input, ensuring efficient and reliable operation of the system. By combining MRAC's self-tuning capability with MPID's precise control, the proposed approach effectively manages the non-linearities inherent in the CT system.

Controlling liquid levels in multiple variations of reference points is shown in Fig. 9. In this figure, the liquid level is varied from 32 cm to 62 cm and the response of all controllers is measured. The proposed MRAC-MPID controller shows better adaptability and allows more precise tracking of varying reference points than the existing controllers. The PID controller shows significant overshoot and longer settling times as it cannot adapt to the system's non-linear dynamics. The FOPID controller performs better than the PID controller due to its fractional-order terms, which offer a certain degree of adaptability. However, it still lags behind the MRAC-MPID in terms of overshoot and response speed because it cannot dynamically tune itself to changing reference points like the MRAC-MPID.

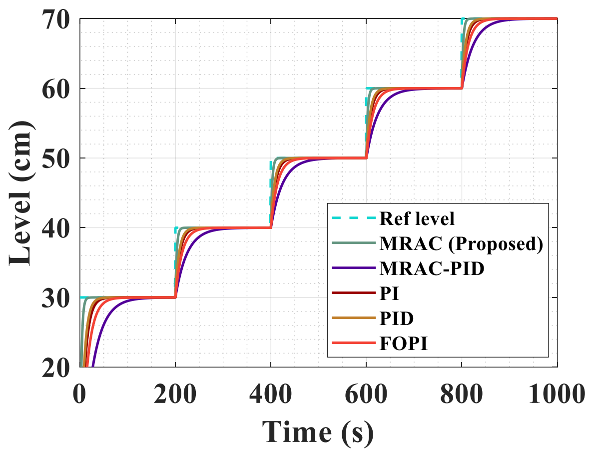

Fig. 9. Controller response under varying liquid levels.

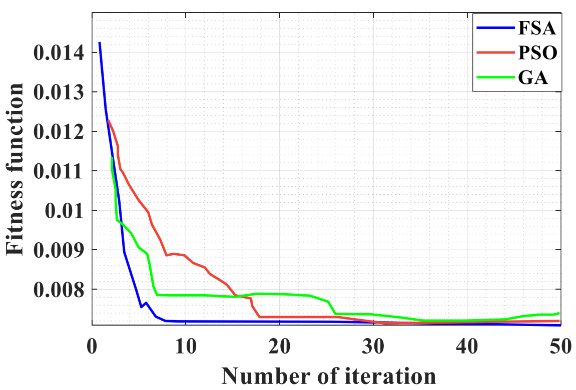

Fig. 10. Convergence plot for optimization algorithms.

As observed, the proposed MRAC outperforms the existing controllers under different operating conditions. The convergence plot of FSA with GA and PSO is shown in Fig. 10. The FSA achieves a more accurate solution with minimal error metrics and ensures that the MRAC-MPID controller is fine-tuned to respond better in non-linear systems such as the CT. From the results, it is verified that the FSA has a better convergence speed than GA and PSO.

Table 3 shows a comparative analysis of the PID controller performance using different optimization techniques, including genetic algorithm (GA), PSO, and FSA as applied in the proposed MRAC-MPID framework. Key performance metrics such as peak overshoot (%), settling time (s), rise time (s), ISE, and IAE are evaluated to highlight the effectiveness of each optimization approach. The manually tuned PID controller exhibits the highest peak overshoot (21.32 %) and the longest settling time (421 s), together with the largest ISE (51.03) and IAE (2968). This shows the limitations of manual tuning in handling the non-linearities and dynamic variations of the CT system. The PID controller optimized with the GA shows significant improvement over manual tuning, with a lower overshoot (10.52 %) and a shorter settling time (376 s). However, the ISE (44.17) and IAE (2888) are still relatively high, indicating room for further improvement in the system's transient and steady-state performance. The use of PSO further improves the controller's performance, reducing overshoot to 5.44 % and shortening the settling time to 305 s. The ISE and IAE also improve to 40.93 and 2657, respectively, showing the effectiveness of PSO in optimizing PID parameters but still leaving some performance gaps. The proposed MRAC-MPID controller optimized with the FSA outperforms all other configurations and achieves the lowest peak overshoot (0.8 %) and settling time (200 s). In addition, the ISE (38.25) and IAE (2167) are significantly lower compared to GA- and PSO-optimized PID controllers. These results highlight the superior ability of the FSA to explore the parameter space and converge to optimal values, resulting in a robust and precise control response. The results emphasize that the FSA effectively minimizes both transient and steady-state errors, resulting in improved system stability and reduced energy consumption. Since the FSA-optimized MRAC-MPID controller accounts for the non-linear dynamics of the CT system more effectively than GA or PSO, it ensures better performance under varying operating conditions and disturbances.

Table 3. Comparative analysis of PID controller with different optimization techniques.

Controllers	Peak overshoot [%]	Settling time [s]	Rise time [s]	ISE	IAE
PID	21.32	421	114	51.03	2968
PID with GA	10.52	376	131	44.17	2888
PID with PSO	5.44	305	122	40.93	2652
MRAC (Proposed)	0.8	200	103	38.25	2167

Fig. 11 shows the comparison of the control signals for different controllers such as PI, PID, FOPI, MRAC-PID, and the proposed method. Table 4 shows the results in terms of control energy and peak values based on the simulation control signals. The control energy values show that the proposed MRAC requires less control energy compared to traditional controllers such as PI, PID, FOPI, and MRAC-PID. This indicates that the proposed MRAC controller is more efficient in terms of energy required to maintain system stability and performance. The peak values show that MRAC controllers have the lowest peak values, indicating that the control signals are smoother and less aggressive compared to the traditional controllers.

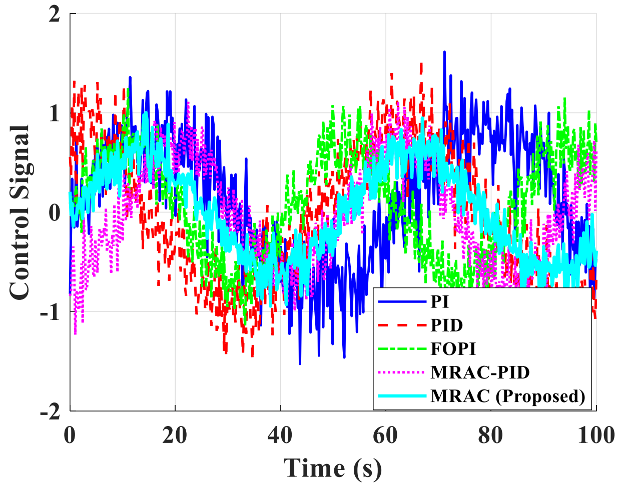

Fig. 11. Comparison of control signals for different controllers.

Table 4. Control energy and peaks for different controllers.

Controller	Control energy [J]	Peak value
PI	41.178	1.6136
PID	50	1.5405
FOPI	28.954	1.2464
MRAC-PID	27.678	1.231
MRAC (Proposed)	20.832	1.002

5. DISCUSSION

The proposed MRAC-MPID controller tuned using the FSA, was evaluated under different scenarios including random disturbances, load variations, and reference point changes. The proposed MRAC-MPID controller shows lower oscillations and faster recovery when subjected to random disturbances in the input flow compared to other controllers. This proves its robustness under dynamic and unpredictable conditions, which is crucial for real-world industrial systems. During load variation scenarios, the MRAC-MPID controller maintains stable operation with minimal impact on liquid levels and exhibits excellent disturbance rejection. This ability to handle varying loads efficiently makes it highly relevant for industries dealing with fluctuating operational demands. The MRAC-MPID controller effectively tracks changing reference points with minimal overshoot and reduced settling times. This proves its adaptability and precise control in dynamic systems that require frequent set point adjustments. The results consistently show that the proposed controller outperforms conventional approaches, such as PI, PID, and FOPI controllers as well as MRAC-PID. Key performance metrics such as lower overshoot, shorter settling time, and better disturbance rejection highlight the controller's ability to effectively handle non-linearities in CT systems. The work focuses on liquid level control in a CT system, although the proposed MRAC-MPID controller is potentially applicable to many non-linear systems and industrial processes. Renewable energy systems, such as wind turbines and solar power plants, are often subject to non-linear dynamics due to environmental fluctuations. The proposed controller could optimize power generation and stabilize system performance with changing wind speeds or solar irradiance levels. Moreover, in the chemical industry, non-linear systems such as Continuous Stirred Tank Reactors and distillation columns show complex dynamic behavior. The proposed MRAC-MPID controller can effectively control parameters such as temperature, concentration, and pressure to ensure the process stability and efficiency. The proposed MRAC-MPID controller is particularly effective in handling system non-linearities, disturbances, and varying operating conditions. However, its performance depends on accurate modeling and proper tuning of the controller. Future research can address these limitations by integrating the method with real-time machine learning models for adaptive tuning in highly uncertain environments.

6. Conclusion

Liquid level control is an important aspect in the process industry with CTs. To improve the liquid level control, an MRAC based controller is proposed in this work. In the proposed MRAC, the MPID is used to improve the level of control under varying inputs and disturbances. The controller gains of MPID are tuned by the FSA based on the *MSE*. The proposed method is implemented in MATLAB/SIMULINK and the results are compared with existing approaches. The results are verified under constant set point, varying set point, and disturbance conditions. It is found that the proposed MRAC has a 0.8 % overshoot and requires 200 s and 103 s for settling time and rising time, respectively. Moreover, the proposed MRAC offers lower ISE and IAE values of 38.25 and 2167, respectively. In addition, the fitness estimation has proved the operating efficiency of FSA compared to existing methods. However, MRAC-MPID outperforms other controllers in handling non-linearities and disturbances and has limitations in terms of its dependence on an accurate reference model. If the reference model does not adequately represent the system dynamics, the controller's performance may deteriorate. While the computational complexity of FSA is lower than that of some optimization methods, it still requires a significant amount of time for parameter tuning in large-scale systems or real-time applications. This could limit the applicability of MRAC-MPID to systems with strict real-time constraints. The proposed method could also face challenges in highly stochastic environments where disturbances are unpredictable and vary significantly over time. These scenarios may require further improvements such as the incorporation of machine learning techniques for adaptive parameter tuning. Future work will develop artificial intelligence-based controllers for two interacting tank systems.

References

[1] Jegatheesh, A., Kumar, C. A. (2020). Novel fuzzy fractional order PID controller for non-linear interacting coupled spherical tank system for level process. *Microprocessors and Microsystems*, 72, 102948. https://doi.org/10.1016/j.micpro.2019.102948

[2] Hadi, M. A., Ali, H. I. (2020). Optimal model reference control scheme design for non-linear strict-feedback systems. *Engineering and Technology Journal*, 38 (9A), 1342-1351. https://doi.org/10.30684/etj.v38i9A.1339

[3] Venkataraman, A. (2021). Design and implementation of adaptive PID and adaptive fuzzy controllers for a level process station. *Advances in Technology Innovation*, 6 (2), 90-105. https://doi.org/10.46604/aiti.2021.6047

[4] Gulzar, M. M., Munawar, M., Dewan, Z., Salman, M., Iqbal, S. (2020). Level control of coupled Conical Tank System using adaptive Model Predictive Controller. In *2020 IEEE 17th International Conference on Smart Communities: Improving Quality of Life Using ICT, IoT and AI (HONET)*. IEEE, 236-240. https://doi.org/10.1109/HONET50430.2020.9322842

[5] Chandrasekar, P., Ponnusamy, L. (2017). Research on variable area hybrid system using optimized Fractional Order Control and Passivity-Based Control. *Computers & Electrical Engineering*, 57, 324-335. https://doi.org/10.1016/j.compeleceng.2016.10.013

[6] Lakshmanan, M., Saravanakumar, N., Kamatchi Kannan, V., Chitra, K., Viji, K., Gowdhamkumar, S. (2021). Real time servo analysis of non-linear conical tank level control using Root Locus Technique. In *2021 Emerging Trends in Industry 4.0 (ETI 4.0)*. IEEE. https://doi.org/10.1109/ETI4.051663.2021.9619399

[7] Aarti, D. S., Arun, N. K. (2021). Liquid level control of quadruple Conical Tank System using Linear PI and Fuzzy PI controllers. In *2021 2nd International Conference for Emerging Technology (INCET)*. IEEE. https://doi.org/10.1109/INCET51464.2021.9456375

[8] Vinothkumar, C., Esakkiappan, C. (2021). Fuzzy PI and Fuzzy PID controller based hopper tank level control system. In *2021 International Conference on Advances in Electrical, Computing, Communication and Sustainable Technologies (ICAECT)*. IEEE. https://doi.org/10.1109/ICAECT49130.2021.9392451

[9] Cruz, F. T., Fernandez, R. D., Guizado, A. G., Zorrilla, J. F. (2021). A comparison of Gain Scheduling PID and μ-Synthesis Robust Level Control for a Conical Tank System. In *2021 IEEE XXVIII International Conference on Electronics, Electrical Engineering and Computing (INTERCON)*. IEEE. https://doi.org/10.1109/INTERCON52678.2021.9532797

[10] Amaguaña, C., Camacho, O. (2021). Sliding mode control for a conical tank: Empirical vs. coordinate transformation linearization comparison. In *2021 IEEE Fifth Ecuador Technical Chapters Meeting (ETCM)*. IEEE. https://doi.org/10.1109/ETCM53643.2021.9590771

[11] Kesavan, S. M., Al Matrushi, F., Al Attar, F., Al Naimi, I., Al Khazraji, A. (2021). Design a heuristic controller for non-linear type liquid level industrial process. In *3rd Smart Cities Symposium (SCS 2020)*. IEEE, 482-486. https://doi.org/10.1049/icp.2021.0880

[12] Febina, C., Vijula, D. A. (2021). RTDA controller design for Conical Tank System. In *2021 International Conference on Advances in Electrical, Computing, Communication and Sustainable Technologies (ICAECT)*. IEEE. https://doi.org/10.1109/ICAECT49130.2021.9392600

[13] Amuthambigaiyin Sundari, K., Maruthupandi, P. (2022). Optimal design of PID controller for the analysis of two tank system using metaheuristic optimization algorithm. *Journal of Electrical Engineering & Technology*, 17 (1), 627-640. https://doi.org/10.1007/s42835-021-00891-6

[14] Patel, H. R., Shah, V. A. (2021). Application of metaheuristic algorithms in interval type-2 fractional order fuzzy TID controller for non-linear level control process under actuator and system component faults. *International Journal of Intelligent Computing and Cybernetics*, 14 (1), 33-53. https://doi.org/10.1108/IJICC-08-2020-0104

[15] Deghboudj, I., Ladaci, S. (2021). Fractional-order multi-model predictive control for non-linear processes. *International Journal of Automation and Control*, 15 (4-5), 611-630. https://doi.org/10.1504/IJAAC.2021.116426

[16] Patel, H. R., Shah, V. A. (2021). Stable fuzzy controllers via LMI approach for non-linear systems described by type-2 T–S fuzzy model. *International Journal of Intelligent Computing and Cybernetics*, 14 (3), 509-531. https://doi.org/10.1108/IJICC-02-2021-0024

[17] Patel, H. R., Raval, S. K., Shah, V. A. (2021). A novel design of optimal intelligent fuzzy TID controller employing GA for non-linear level control problem subject to actuator and system component fault. *International Journal of Intelligent Computing and Cybernetics*, 14 (1), 17-32. https://doi.org/10.1108/IJICC-11-2020-0174

[18] Urrea, C., Páez, F. (2021). Design and comparison of strategies for level control in a non-linear tank. *Processes*, 9 (5), 735. https://doi.org/10.3390/pr9050735

[19] Chen, X., Peng, D., Wu, W., Liu, H., Zheng, X. (2022). Active control of torsional vibration during mode switching of hybrid powertrain based on adaptive model reference. *Machines*, 10 (8), 647. https://doi.org/10.3390/machines10080647

[20] Khelas, S. E., Ladaci, S., Bensafia, Y. (2020). Fractional order adaptive MRAC controller for an active suspension system: Fractional operator, fractional order system, Model Reference Adaptive Control, active suspension system. *Algerian Journal of Signals and Systems*, 5 (2), 112-117. https://doi.org/10.51485/ajss.v5i2.105

[21] Sahaya Aarti, D., Arun, N. K. (2022). Gain scheduled adaptive Model Predictive Controller design for Quadruple Conical Tank System. In *2022 3rd International Conference for Emerging Technology (INCET)*. IEEE. https://doi.org/10.1109/INCET54531.2022.9824024

[22] Kumar, M., Prasad, D., Singh, R. S. (2023). Level control in conical tank using IMC-PID controller. *Journal of Engineering Science and Technology Review*, 16 (2), 71-81. http://doi.org/10.25103/jestr.162.10

[23] Ranjan, R. (2021). Robust Model Reference Adaptive Control for liquid level control in process industry. *International Journal of Advances in Engineering and Management (IJAEM)*, 3 (4), 81-88.

[24] Espitia-Cuchango, H., Machón-González, I., López-García, H. (2022). Filling control of a conical tank using a compact neuro-fuzzy adaptive control system. *Complexity*, 2022, 4284378. https://doi.org/10.1155/2022/4284378

[25] Aguila-Camacho, N., Farías-Ibañez, S. E. (2022). Level control of a Conical Tank System using switched fractional order PI controllers: An experimental application. In *2022 10th International Conference on Control, Mechatronics and Automation (ICCMA)*. IEEE, 82-87. https://doi.org/10.1109/ICCMA56665.2022.10011620

[26] Rajesh, R. J., Preethi, R., Mehata, P., Pandian, B. J. (2015). Artificial neural network based inverse model control of a non-linear process. In *2015 International Conference on Computer, Communication and Control (IC4)*. IEEE. https://doi.org/10.1109/IC4.2015.7375581

[27] Rajesh, R. (2019). Optimal tuning of FOPID controller based on PSO algorithm with reference model for a single conical tank system. *SN Applied Sciences*, 1, 758. https://doi.org/10.1007/s42452-019-0754-3

[28] Wang, Z., Liu, J. (2021). Flamingo search algorithm: A new swarm intelligence optimization algorithm. *IEEE Access*, 9, 88564-88582. https://doi.org/10.1109/ACCESS.2021.3090512

[29] Balaska, H., Ladaci, S., Zennir, Y. (2018). Conical tank level supervision using a fractional order model reference adaptive control strategy. In *Proceedings of the 15th International Conference on Informatics in Control, Automation and Robotic (ICINCO)*, vol. 1, 224-231. https://doi.org/10.5220/0006869602140221

[30] Patil, S. R., Agashe, S. (2024). Deep Reinforcement Learning Controller for Conical Tank Process. In *2023 Second IEEE International Conference on Measurement, Instrumentation, Control and Automation (ICMICA)*. IEEE. https://doi.org/10.1109/ICMICA61068.2024.10732564

[31] Ramanathan, P., Mangla, K. K., Satpathy, S. (2018). Smart controller for conical tank system using reinforcement learning algorithm. *Measurement*, 116, 422-428. https://doi.org/10.1016/j.measurement.2017.11.007

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