Dynamic Structural Equation Modeling (DSEM) is an advancement within the framework of Structural Equation Modeling (SEM) and allows for the modeling of multilevel data, integrating time series analysis with SEM. It is based on a Bayesian framework, with parameters estimated through a Markov Chain Monte Carlo process. DSEM is particularly effective in handling intensive longitudinal data (Schultzberg et al., 2017; Blanke et al., 2022).

In this study, DSEM provided the necessary bidirectional modeling type to study the reciprocal effects of rumination and self-esteem over time. Rumination was modeled as the independent variable, and self-esteem as the dependent variable, with time included as a control variable to account for linear trend effects. The model was divided into within-group and between-group levels.

Time was encoded according to the order of the completed questionnaires' dates, for example: the first day was coded as "1", the second day as "2", and so on. Rumination, self-esteem, and time were group-mean centered. The specific model equations are as follows. At the within-group level, the self-esteem y_{ij} of individual *i* on day *j* can be represented as:

Within-group level: $y_{ij} = \beta_{0i} + \beta_{1i}(X_{ij} - X) + \beta_{2i}(T_{ij} - T) + r_{ij}$

In the equation, β_{0i} represents the intercept of self-esteem, which is the average self-esteem value for participant *i*. β_{1i} and β_{2i} represent the slopes, indicating the rate at which rumination predicts changes in self-esteem, and the rate at which time predicts changes in self-esteem, respectively, for participant *i*. X_{ij} and X represent the level of rumination for participant *i* on day *j* and the average level of rumination for individual *i*, respectively. T_{ij} and T represent the measurement time and the average measurement time. r_{ij} denotes the error term, representing the part of the measurement value *y* for participant *i* at observation time *j* that cannot be explained by the independent variables.

For the between-group level, the model expression is as follows: Between-group level: $\beta_{0i} = \gamma_{00} + u_{0i}$; $\beta_{1i} = \gamma_{01} + u_{1i}$; $\beta_{2i} = \gamma_{02} + u_{2i}$

In the model, β_{0i} is the random intercept at Level 1, and β_{1i} and β_{2i} are the random slopes at Level 1. γ_{00}, γ_{01}, and γ_{02} represent the corresponding fixed intercepts, and u_{0i}, u_{1i}, and u_{2i} are the residuals for the corresponding equations.

To explore the causal predictive relationship between daily rumination and daily self-esteem, we constructed a multilevel cross-lagged path model. As shown in Figure S1, γ_{1j} and γ_{4j} represent autoregressive effects, while γ_{2j} and γ_{3j} represent cross-lagged effects. $\mu_{rumination}$ and $\mu_{self\text{-}Esteem}$ are the intercepts for rumination and self-esteem, respectively. In this model, both intercepts and slopes are set as random. To keep the model concise, the equations for the within-group level analysis are not presented here. The specific model equation is as follows: γ_{ij} *(Rumination following day)* $= \gamma_{0j} + \gamma_{1j}$ *(Rumination previous day)* $+ \gamma_{2j}$ *(Self-esteem previous day)* $+ r_{ij}$; γ_{ij} *(Self-esteem following day)* $= \gamma_{0j} + \gamma_{3j}$ *(Rumination previous day)* $+ \gamma_{4j}$ *(Self-esteem previous day)* $+ r_{ij}$.

In the model, γ_{1j} *(Rumination Previous Day)* represents the slope at which the previous day's rumination of participant i predicts their rumination on the following day. γ_{2j} *(Self-esteem Previous Day)* represents the slope at which the previous day's self-esteem of participant *i* predicts their rumination on the following day. γ_{3j} *(Rumination Previous Day)* represents the slope at which the previous day's rumination of participant *i* predicts their self-esteem on the following day. γ_{4j} *(Self-Esteem Previous Day)* represents the slope at which the previous day's self-esteem of participant *i* predicts their self-esteem on the following day.

This multilevel cross-lagged model facilitates the understanding of how rumination and self-esteem interact over time within individuals, providing insight into the dynamic interplay between these two psychological constructs and their influence on each other from one day to the next.

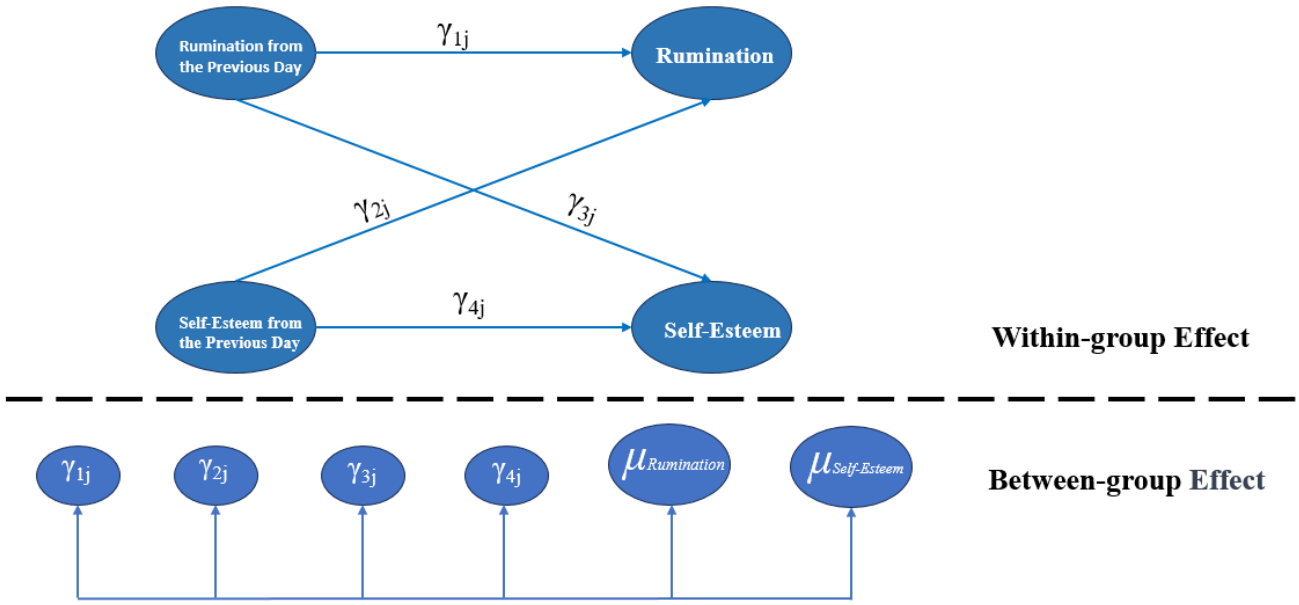

Figure S1 Two-level regression model diagram of daily rumination and daily self-esteem.

References

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